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OPEN Large waves and navigation hazards of the Eastern Mediterranean Sea

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We present a statistical analysis of deep-water buoy measurements of large waves generated during two major storms of the Eastern Mediterranean in 2017 and 2018, respectively. The largest waves observed do display similar characteristics to those of the Draupner, Andrea, and El Faro roque waves in that second order bound nonlinearities enhance the linear dispersive focusing of extreme waves. We also present a novel analysis of waves in space-time to predict potential risks posed by such large waves to navigation. In particular, we consider the scenario of two types of vessels of the Israeli Navy fleet navigating during the most intense stages of the two storms considered here and provide predictions for the largest waves likely to be encountered.

In light of global warming, weather events are likely to become more severe and hazardous to coastal and offshore structures and maritime navigation. Not only does this relate to vast oceans, but also to large basins such as the Black Sea and the Mediterranean Sea. In particular, the Eastern Mediterranean deserves special attention because it lacks studies on the physical characteristics of extreme sea states and large waves. A climatology hotspot and natural treasures area, the Eastern Mediterranean is a basin of increasing interest, both ecologically and economically¹. Thus, a scientific investigation and prediction of wave extremes are of paramount importance, especially for rogue wave predictions and maritime navigation in extreme weather.

Conventional thresholds² have been adopted to distinguish if a wave is rogue or not. In particular, a wave is rogue if its crest height $h > 1.25H_s$ or the crest-to-trough (wave) height $H > 2H_s$, where H_s is the significant wave height of the sea state in which the rogue wave occurred. For example, the first famous rogue wave ever recorded is the Draupner wave that occurred on the 1st of January 1995 in an intense sea state with $H_s = 11.9 \,\mathrm{m}^3$, 4. The 26-m rogue wave hit the lower deck of the "Draupner S" oil platform at the North Sea. The Draupner wave has $H = 2.15H_s$ and $h = 1.55H_s$ satisfying the conventional criteria for being rogue². On November 2007, the powerful Andrea storm crossed the "Ekofisk" oil platform. In a sea state with $H_s = 9.2$ m four wave gauge lasers mounted on the rig measured the Andrea rogue wave⁵, whose wave height $H = 2.3H_s$ and crest height $h = 1.62H_s$. In November 2020 a rogue wave has been detected off the Vancouver Island in the Atlantic Ocean⁶, whose wave height $H = 2.92H_s$ and crest height $h = 1.98H_s$ are well over the conventional rogue thresholds². Nevertheless, the crest height of that rogue wave reached 11.96 m in amplitude, and thus smaller than both the Draupner and Andrea waves.

There is a lack of scientific knowledge about the metocean characteristics of the eastern part of the Mediterranean Sea, especially for rogue wave predictions and vessel navigation in extreme weather. Reports of rogue wave occurrences involving vessel navigation have been mostly observed in the western part of the Mediterranean by eyewitnesses on cruise ships that have been impacted by extreme waves resulting in structural damage and loss of lives. Two cases have been recently reported: the "Voyager" accident that occurred in 2005 and the "Louis Majesty" accident in 2010. Both accidents triggered scientific interest to confirm, or rebut the possibility of cruise ships encountering rogue waves^{7,8}. A rogue wave event has been caught in the Northern Adriatic Sea by a web-camera placed on the oceanographic tower 'Acqua Alta' off the Venice coast, Italy. The estimated wave crest height is in the range of 5.1-6.4 m⁹ and exceeded the conventional rogue thresholds². All the above-mentioned extreme waves have been observed in the western part of the Mediterranean Sea.

Recently, Knobler et al. 10 provided the first observations of several rogue wave events in the Eastern part of the Mediterranean Sea from measurements acquired by AXYS Technologies "3 metre" metadata buoy in deep

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	El Faro	Andrea	Draupner	Jan 2017	Gaia
Significant wave height H _s [m]	9.0	10.0	11.2	6.86	6.16
Dominant wave period T_p [s]	10.2	14.3	15.0	11.9	14.1
Mean zero-crossing wave period T_0 [s]	9.2	11.6	11.9	8.6	9.2
Mean wavelength L_0 [m]	131	209	219	115	131
Depth d [m], $k_0 d$ with $k_0 = 2\pi/L_0$	4700, 2.25	74, 2.23	70, 2.01	258,14.11	258, 12.36
Spectral bandwidth ν	0.49	0.35	0.36	0.43	0.46
Angular spreading σ_{θ} [rad]	0.79	0.37	0.39	1.08	1.11
Parameter $R = \sigma_{\theta}^2 / 2\nu^2$	1.34	0.56	0.59	3.18	2.88
Benjamin Feir Index BFI in deep water ¹⁷	0.36	0.24	0.23	0.16	0.09
Tayfun NB skewness $\lambda_{3,NB}^{18}$	0.262	0.159	0.165	0.222	0.199
Observed skewness λ_3	0.162	0.141	0.146	0.103	0.105
Maximum NB dynamic excess kurtosis $\lambda_{40,max}^{d}$ 19	10^{-3}	2.3×10^{-3}	2.1×10^{-3}	-4.8×10^{-3}	-1.6×10^{-3}
Janssen NB bound excess kurtosis $\lambda_{40,NB}^d$ 20	0.049	0.065	0.074	0.097	0.087
Observed excess kurtosis λ_{40}	0.042	0.041	0.032	0.100	0.087
Actual maximum crest height h/H _s	1.68	1.63	1.55	1.04	1.11
Actual maximum crest-to-trough (wave) height H/H_s	2.6	2.30	2.10	1.98	1.96

Table 1. Wave parameters and various statistics of Jan 2017 and Gaia at the storm peak in comparison to the El Faro, Andrea and Draupner rogue sea states¹². We refer to the "Methods" section for the definitions of wave parameters.

waters (see "Methods" section for details). Surface elevations, wind and wave parameters were also collected by a meteorological station ^{10, 11}. As expected for this region, the strongest storm events occurred throughout October–April period ¹⁰. They detected a sample of 109 rogue waves, whose wave heights exceeded the rogue threshold $2H_s$, and only nine waves out of the sample had crests exceeding $1.25H_s$ in height². However, no wave crest exceeded 4 m in height.

In the present work, we further examine the statistics of large waves in deep waters observed during the two most powerful storm events of the Eastern Mediterranean recorded during the last five years. In particular, the first storm occurred during January 16–22 in 2018, and hereafter referred to as Gaia. The second storm occurred during January 25–31 in 2017, and hereafter referred to as Jan 2017. We note that the largest waves observed in both storms do not exceed the rogue wave thresholds², in contrast to the Draupner and Andrea rogue waves^{3–5, 12, 13} that occurred in intermediate water depths and did reach crest heights of $1.6H_s$ well over the rogue threshold. In our studies, we also exploit statistical distributions for non-stationary time series¹⁴ to analyze the statistics of surface elevations sampled from the two storms. Finally, we carry out a novel space-time analysis of the most intense sea states as a basis for predicting potential rogue hazards for ships navigating through the rough waters of the storms.

Results

This section is structured as follows. First, we discuss the metocean characteristics of the sea states generated by the two observed storms as they passed through the Eastern Mediterranean. Relevant wave parameters and statistical models are defined in the "Methods" section. Then, we make prediction on the extreme waves during the two storms. To do so, we draw on Fedele et al. ¹⁴ and use their formulation of the probability structure of non-stationary time series as a model for the sequence of sea states in a storm. In particular, we study the occurrence frequencies of rogue waves during Jan 2017 and Gaia storms encountered by an observer at a given point of the sea surface. The wave extremes observed at the peak of the storms and their characteristics are then compared to those of the Andrea and Draupner rogue waves, observed at different oil platforms in the North Sea in 1995 and 2007, respectively¹² as well as the simulated El Faro rogue wave¹³. The metocean parameters of these five sea states are summarized in Table 1. Finally, we present a space-time wave analysis^{13, 15, 16} of the most intense sea states to predict rogue hazards for vessel navigation within an area.

Our extreme wave analysis focused on the study of the time sequence of changing sea states during the two storms. While the analysis of Fedele et al. ¹⁴ yields an optimal sea state duration T_{sea} of 40–50 min to minimize the variation between waves of consecutive sea states of storms, the duration T of our actual measurements was limited to 18 min at the beginning of each hour to minimize data storage. So, the analyses here were unavoidably based on $T_{sea} = T = 18$ min. Nevertheless, we observed that our wave statistics are very robust to changes in T_{sea} up to ± 5 min, provided that surface elevations of each sea state in the storm sequence are normalized by the respective significant wave height.

Metocean parameters. The metocean parameters history of Jan 2017 storm are shown in Fig. 1. In particular, the top panel depicts the hourly variation of the significant wave height. This is estimated as $H_s = 4\sigma$ (black line). For comparison, $H_{1/3}$ is also shown (red line), which is 5% smaller than H_s . The mean zero-upcrossing wave period T_0 is shown in the second panel of the same figure, and the hourly variations of $k_p d$ are reported in the third panel, where k_p is the peak wavenumber. The bottom panel on the left depicts the frequency

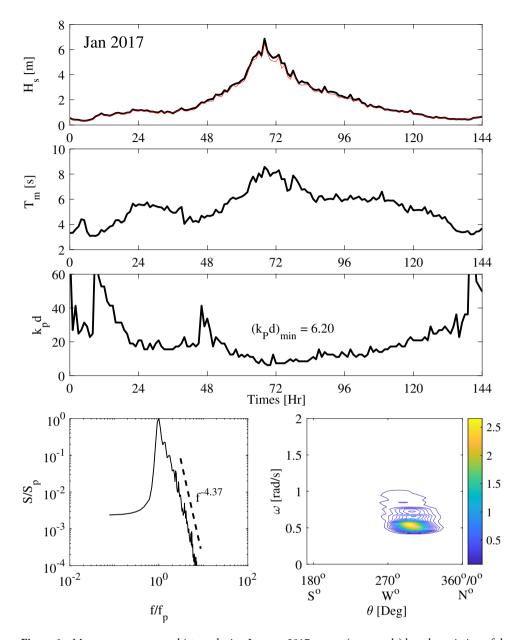


Figure 1. Metocean parameters history during January 2017 storm: (top panels) hourly variation of the significant wave height estimated as $H_s = 4\sigma$ (black) and $H_{1/3}$ (red), mean zero-up-crossing wave period T_0 , and depth factor $k_p d$; (bottom-left) normalized frequency spectrum $S(f)/S_p$ and (bottom-right) the directional spectrum $S(\omega,\theta)$ ([m²/rad/s/deg]) at the storm peak. Here, S_p ([m²/Hz]) and f_p ([Hz]) are the spectral amplitude and frequency at the storm peak.

spectrum measured at the storm peak. The directional wave spectrum is also shown, in the bottom panel on the right. Similarly, the histories of metocean parameters of storm Gaia are presented in Fig. 2. The two storms generated waves in deep waters depth $(k_pd>\pi,\sec^{14})$. Moreover, k_pd is well above the critical threshold 1.363, at which modulation instability disappears. Nevertheless, third-order nonlinear modulational effects are negligible because the energy of the two sea states spreads both in all directions and frequencies 12,19 as clearly seen in the bottom panels of Figs. 1, 2. Furthermore, we depict in Fig. 3 the Boccotti parameter ψ^* , which measures the spectral bandwidth of the sea states of the two storms. The values of the observed ψ^* vary in the range [0.5-0.7] indicating sea states with broadband spectra. In Table 1 we report the metocean parameters of the two storm events at their peak stages in comparison to those of the El Faro, Draupner and Andrea rogue sea states 12,13 . Note that the five sea states all have similar metocean characteristics. Both Jan 2017 and Gaia are in deep waters and developed comparable steep sea states as the other three sea states, even if their peak significant wave height is approximately 30% smaller than that of the other rogue sea states. Indeed, the observed wave parameters have comparable values in the same range typical of oceanic sea states in storms. Our analysis indicates that the largest observed waves in Gaia and Jan 2017 storms have characteristics quite similar to those displayed by the El Faro,

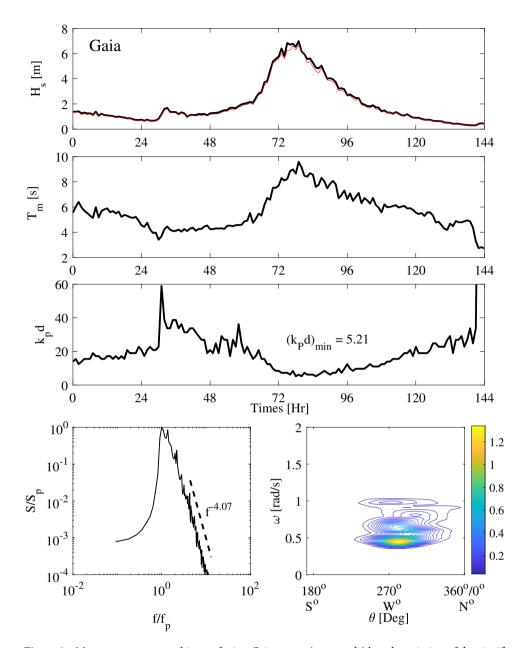


Figure 2. Metocean parameters history during Gaia storm: (top panels) hourly variation of the significant wave height estimated as $H_s = 4\sigma$ (black) and $H_{1/3}$ (red), mean zero-up-crossing wave period T_0 , and depth factor $k_p d$; (bottom-left) normalized frequency spectrum $S(f)/S_p$ and (bottom-right) the directional spectrum $S(\omega,\theta)$ ([m²/rad/s/deg]) at the storm peak. Here, S_p ([m²/Hz]) and f_p ([Hz]) are the spectral amplitude and frequency at the storm peak.

Andrea and Draupner rogue waves^{12, 13}, where second order bound nonlinearities are the dominant factor that enhances the linear dispersive focusing of extreme waves.

Statistical measures of ocean nonlinearities. We use integral statistics such as the Tayfun steepness $\mu = \lambda_3/3$ and the excess kurtosis λ_{40} of the zero-mean surface elevation $\eta(t)$ to measure ocean wave nonlinearities. Wave skewness λ_3 describes the effects of second-order bound nonlinear effects, which alter the geometry and statistics of the sea surface with higher sharper crests and shallower more rounded troughs^{23–25}. The excess kurtosis indicates deviations of the distribution of surface elevations from Gaussianity. It comprises a dynamic component^{19, 26} λ_{40}^d measuring third-order quasi-resonant wave-wave interactions and a bound contribution λ_{40}^b induced by both second- and third-order bound nonlinearities^{23–28}.

Figure 4 depicts the statistical parameters history for the two analyzed storms. The top panels display the hourly variations of the observed Tayfun steepness $\mu = \lambda_3/3$, the observed excess kurtosis λ_{40} and the theoretical maximum Narrow Band (NB) dynamic excess kurtosis 19 $\lambda_{40,max}^d$ for storm Jan 2017. The theoretical

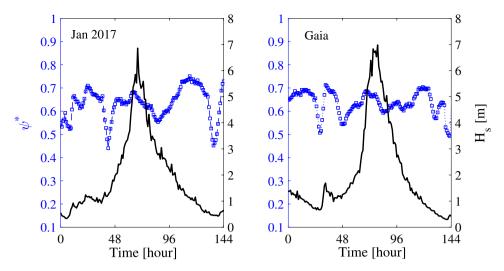


Figure 3. Metocean parameters history during storms Jan 2017 (left panel, start date Jan 25, 2017, 00:00) and Gaia (right panel, start date Jan 16, 2017, 00:00): hourly variation of Boccotti parameter ψ^* and the significant wave height estimated as $H_s = 4\sigma$.

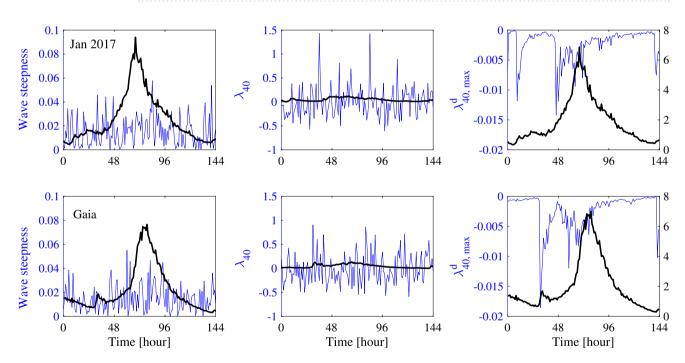


Figure 4. Statistical parameters history during storms (top panels) Jan 2017 and (bottom panels) Gaia: (left) hourly variation of the theoretical NB (bold line) and observed Tayfun steepness $\mu = \lambda_3/3$ (thin blue line), (center) observed excess kurtosis (thin blue line) and NB estimate (bold line) and (right) theoretical maximum NB dynamic excess kurtosis¹⁹ (blue thin line). The significant wave height history is also reported (black line).

narrow-band predictions are also shown. The same comparisons for storm Gaia are also reported in the bottom panels. In Table 1 we compare the statistical parameters of Gaia and Jan 2017 strongest sea states and the Draupner and Andrea rogue sea states (from Fedele et al. 14). All sea states are short-crested as characterized by large directional spreading, especially at the peak of the storms as R>1. In open ocean, wave energy can spread directionally and third-order quasi-resonant nonlinearities are essentially insignificant $^{12,19,29-32}$ in such realistic oceanic conditions 12,33,34 . Indeed, the right panels of Fig. 4 indicate that the maximum dynamic excess kurtosis is of $O(10^{-3})$ and negative for both Jan 2017 and Gaia, and negligible in comparison to the associated bound component. The values of the observed excess kurtosis from the hourly 18 min samples are erratic due to the short record duration. Estimates of the third order moment, or wave skewness from the buoy's time records of surface elevations converged for time series 70–100 wave periods long, or 12–15 min records. In contrast, estimates of the fourth-order moment, or excess kurtosis were at the verge of convergence for the available 18-min record. Nevertheless, our analysis indicates that third-order quasi-resonant interactions, including NLS-type

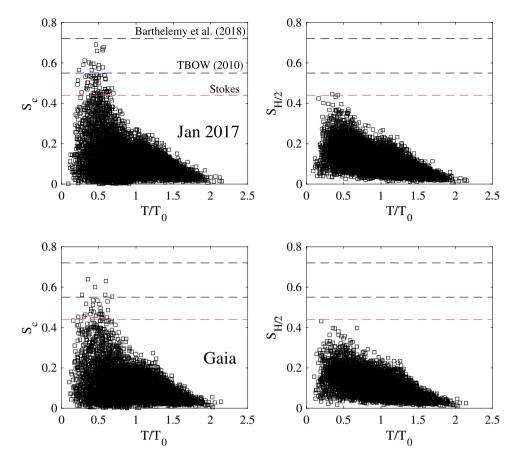


Figure 5. Observed Local steepnesses $S_c = kh$ and $S_{H/2} = kH/2$ versus local period T/T_0 for (top panels) Jan 2017 and (bottom panels) Gaia storms. Stokes limit = 0.448, TWBO limit²¹ = 0.55 and Barthelemy et al.²² = 0.72. k = local wave number, h = crest height, H = crest-to-trough height and T_0 = mean zero-crossing period. Because of the crest-trough asymmetry, on average h > H/2 and $S_c > S_{H/2}$.

modulational instabilities play an insignificant role in the formation of large waves 12,19 in agreement with oceanic observations available so far 24,35,36 .

Next we analyze the local steepness of the individual waves observed. To do so, we consider the two definitions $S_c = kh$ and $S_{H/2} = kH/2$ based on the crest height h and the half of the crest-to-trough height H, respectively. Here, k is the wavenumber estimated from the measured local wave period using the nonlinear dispersion relation^{21,37}. Toffoli et al.²¹ used $S_{H/2}$ to analyze the statistics of individual wave steepnesses from both laboratory and open sea measurements. They concluded that waves can reach $S_{H/2}$ values above the Stokes limit 0.448 and up to the limit 0.55, hereafter referred to as TBOW. Clearly, H/2 is smaller than the crest height h on average because of the crest-trough asymmetry due to bound nonlinearities²³. As a consequence $S_{H/2}$ is smaller than S_c . Indeed, in a numerical study of breaking wave groups Barthelemy et al.²² observed $S_c = kh$ values that did not exceed the limit 0.72, which is based on the fifth order Stokes wave solution³⁸. Recently, Knobler et al.³⁹ observed in their experiments waves of average amplitude with steepness S_c above 0.8. The steepest waves appear as forms of wave dislocations due to crest pairing, or splitting⁴⁰. However, the largest waves in their dataset did not exceed the theoretical limit²² 0.72. Figure 5 depicts the local steepness of the individual waves measured during Jan 2017 and Gaia storms. Note that the observed $S_{H/2}$ values do not exceed the TBOW limit 0.55. Similarly, the observed S_c values do not exceed the 0.72 limit proposed by Barthelemy et al.²².

Occurrence frequency of a rogue wave during storms. We now describe the statistics of large waves encountered by an observer at a fixed point of the ocean surface during a storm of duration T_s . To do so, we draw on Fedele et al. 14 and model the storm as a non-stationary continuous sequence of sea states of duration dt, and $dt/T_0(t)$ is the number of waves in the sea state, where $T_0(t)$ is the time-dependent mean zero-crossing wave period. We want to estimate the probability, or occurrence frequency $P_{ns}(\xi)$ of a wave crest height exceeding a given threshold ξH_s as encountered by a fixed observer, where $H_s = 4\sigma$. That is the probability to randomly pick from the non-stationary time series observed at a fixed point a wave crest that exceeds the threshold ξH_s , and it is given by 14

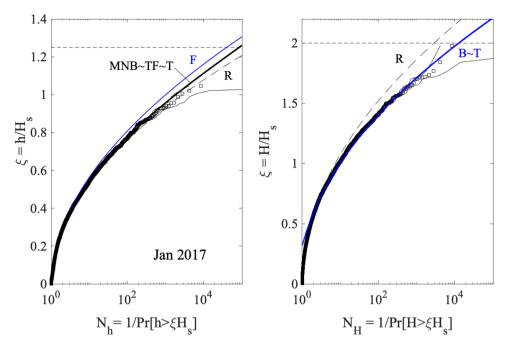


Figure 6. Wave statistics for storm Jan 2017: (left) crest heights h/H_s and (right) crest-to-trough wave heights H/H_s versus number of waves $N_h(\xi)$. The empirical distribution (squares) from the observed wave ensemble is compared against with theoretical models: T = Tayfun^{23, 25}, TF = Tayfun-Fedele²⁴, MNB = modified narrow-band⁴¹, B = generalized Boccotti⁴², F = Forristall⁴³ and R = Rayleigh distributions. Confidence bands are also shown (light dashes). $N_h(\xi)$ is the inverse of the exceedance probability $\Pr[h > \xi H_s]$, and similarly $N_H(\xi)$ is for crest-to-trough heights. Horizontal lines denote the rogue threshold 1.25 H_s and 2 H_s for crest and wave heights².

$$P_{ns}(\xi) = \frac{\int_0^{T_s} P(\xi = h/H_s(t), t) \frac{1}{T_0(t)} dt}{\int_0^{T_s} \frac{1}{T_0(t)} dt},$$
(1)

where $P(\xi,t)$ is the stationary exceeding probability of the sea state occurring in the time interval t and t+dt, which depends on wave parameters around time t. The definition of P_{ns} is consistent with the way wave crests are sampled from non-stationary wave measurements during storms. Moreover, the threshold ξH_s is exceeded on average once every $N_h(\xi) = 1/P_{ns}(\xi)$ waves, where N_h is also the conditional return period of a wave whose crest height exceeds ξH_s . In practice, the non-stationary P_{ns} is estimated as the weighted average¹⁴

$$P_{ns}(\xi) = \frac{\sum_{j=1}^{N_s} P(\xi = h/H_s(t_j)) N_{w,j}}{\sum_{j=1}^{N_s} N_{w,j}},$$
 (2)

where $N_{w,j}$ is the number of waves sampled in the sea state j and a storm is partitioned in a finite sequence of N_s sea states of duration D_s . For weakly nonlinear random sea states (stationary seas), the probability P is hereafter described by the third order Tayfun-Fedele²⁴ (TF), modified narrow-band⁴¹ (MNB), Tayfun^{23, 25} (T), Forristall⁴³ (F) and the linear Rayleigh (R) distributions (see "Methods" section). Similarly, the non-stationary occurrence frequency $P_{ns}(H)$ of a wave in a storm whose crest-to-trough (wave) height exceeds the threshold H/H_s are described by the same Eqs. (1) by simply replacing P(h) with the exceedance probability P(H) proper for wave heights of stationary seas. This is hereafter described by the generalized Boccotti⁴² (B), Tayfun⁴⁴ (T) and linear Rayleigh (R) distributions (see "Methods" section).

Figure 6 summarizes the wave statistics for storm Jan 2017. In particular, the left panel depicts the empirical distribution (squares) for crest heights h/H_s plotted versus the number of waves $N_h(\xi)$. This is compared against theoretical predictions of the non-stationary second-order F, MNB and T models as well as the third-order TF and Rayleigh (R) distributions. T and TF models are based on the observed Tayfun steepness $\mu = \lambda_3/3$. Note that TF is practically the same as MNB as an indication that second-order effects are dominant. F slightly overestimates the return periods, whereas the linear R model underestimates them. This indicates the dominance of second-order nonlinearities in shaping the sea surface. Similarly, the right panel of the same Fig. 6 reports the empirical distribution for crest-to-trough wave heights for Jan 2017. The observed statistics is well described by both the generalized Boccotti (B) and Tayfun (T) models. Similar conclusions also hold for the wave statistics for Gaia presented in Fig. 7.

The wave profile η with the largest wave crest height observed during Gaia ($h=1.11H_s\approx 6.8$ m, $H_s=6.16$ m) is shown in the right panel of Fig. 8. For comparison, the El Faro, Draupner and Andrea rogue wave profiles are also shown^{12, 13}. From Table 1, Gaia wave did not exceed the rogue threshold $1.25H_s$ as the other three rogue

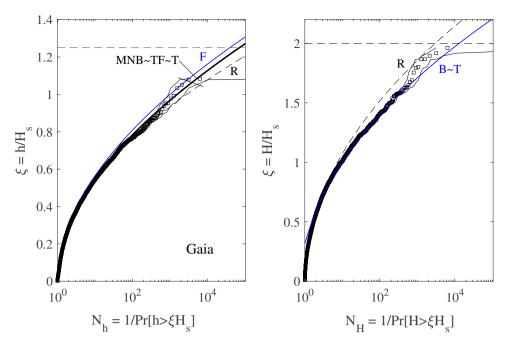


Figure 7. Wave statistics for storm Gaia: (left) crest heights h/H_s and (right) crest-to-trough wave heights H/H_s versus number of waves $N_h(\xi)$. The empirical distribution (squares) from the observed wave ensemble is compared against with theoretical models: $T = Tayfun^{23,25}$, TF = Tayfun-Fedele²⁴, MNB = modified narrow-band⁴¹, B = generalized Boccotti⁴², F = Forristall⁴³ and R = Rayleigh distributions. Confidence bands are also shown (light dashes). $N_h(\xi)$ is the inverse of the exceedance probability $Pr[h > \xi H_s]$ for crest heights, and similarly $N_H(\xi)$ is for crest-to-trough heights. Horizontal lines denote the rogue threshold $1.25H_s$ and $2H_s$ for crest and wave heights².

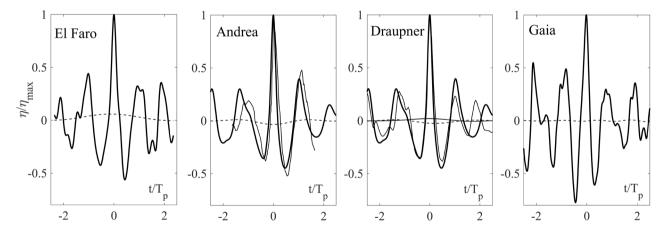


Figure 8. Simulated wave profiles η/η_{max} (solid) and mean sea levels (MSL) (dashed) versus t/T_p for (from left to right) El Faro, Andrea and Draupner sea states. Gaia largest wave observed is shown in the right panel. Actual measurements (thin solid) and MSLs (thin solid) are also shown for Draupner. Note that the Gaia MSL is insignificant and the Andrea MSL is not available. η_{max} is the maximum crest height and T_p is the dominant wave period (see Table 1 and "Methods" section for definitions).

waves did. Nevertheless, the five sea states have very similar characteristics, suggesting a common generation mechanism due to linear dispersive focusing enhanced by second-order nonlinearities¹².

Wave statistics in space-time. The maximum surface wave height observed over a given area during a time interval, or space-time extreme is much larger than the maximum observed at a given point over the same time period^{15, 16, 48}. Indeed, in the short-crested directional seas typically generated by storms it is very unlikely that an observed large crest at a given point in time is actually the largest crest of a group of waves propagating in space-time. In contrast, it is most likely that the sea surface was much higher somewhere near the measurement point, as Boccotti's (2000) QD theory predicts.

Euler Characteristics of random fields^{49–51} provide a theoretical framework for modelling space-time wave extremes stochastically^{15, 16, 52}. Drawing on Fedele et al.¹³ consider a 3-D non-Gaussian field $\eta(x,y,t)$ in space-time over an area A for a time period of D. The area cannot be too large since the wave field may not be statistically homogeneous. The duration should be short so that the sea state can be assumed as stationary and spectral changes insignificant. Then, drawing on the linear Gaussian space-time model^{49–51} the third-order nonlinear probability $P_{\rm ST}(\xi;A,D)$ that the maximum surface elevation $\eta_{\rm max}$ over the area A and during the time interval D exceeds the generic threshold ξH_s is described by $^{13, 49–51}$

$$P_{\rm ST}(\xi; A, D) = \Pr\{\eta_{\rm max} > \xi H_s\} = (16M_3 \xi_0^2 + 4M_2 \xi_0 + M_1) P_{\rm TF}(\xi), \tag{3}$$

where

$$P_{\text{TF}}(\xi) = \exp\left(-8\xi_0^2\right) \left[1 + \Lambda \xi_0^2 (4\xi_0^2 - 1)\right],\tag{4}$$

denotes the third-order nonlinear Tayfun-Fedele model²⁴ for the probability of exceedance of crest heights at a fixed point. The parameter Λ measures third-order nonlinear effects and can be estimated from the excess kurtosis as $\Lambda_{appr} = 8\lambda_{40}/3$ (see "Methods" section). Note that our analysis indicates that third-order nonlinearities are negligible. The amplitude ξ relates to the Rayleigh-distributed envelope ξ_0 via the Tayfun quadratic equation^{11, 23, 25}

$$\xi = \xi_0 + 2\mu \xi_0^2. \tag{5}$$

The space-time parameters M_1 and M_2 are the average number of 1-D and 2-D waves that can occur on the edges and boundaries of the volume Ω , and M_3 is the average number of 3-D waves that can occur within the volume 15. These all depend on the directional wave spectrum and its spectral moments defined in the "Methods" section.

Given the probability structure of the linear surface wave maximum in Eq. (3), the nonlinear mean maximum surface or crest height $\bar{h}_{ST} = \xi_{ST} H_s$ attained over the area A during a time interval D is given, according to Gumbel (1958), by¹³

$$\xi_{\rm ST} = \overline{h}_{\rm ST}/H_{\rm s} = \xi_{\rm m} + \gamma_e \beta,\tag{6}$$

where $\beta=1/P_{ST}'(\xi)$ is the inverse of the derivative of $P_{ST}(\xi)$ with respect ξ , the Euler-Mascheroni constant $\gamma_e\approx 0.577$, and the most probable surface elevation value $\xi_{\rm m}$ follows from Eq. (3) as

$$P_{\rm ST}(\xi_{\rm m}; A, D) = 1. \tag{7}$$

Fedele et al.¹³ observed that Gaussian space-time models^{53–57} overestimate the maximum surface height over large areas and time intervals because they rely on Gaussianity. Indeed, Gaussian seas do not account for the wave physics of nonlinear dispersion²⁹, which limits the wave growth as a precursor to breaking^{29,58}. As a result, water surface elevations of a Gaussian sea can reach any height as there are no physical limits induced by wave breaking. Thus, the larger the area A or the time interval D, the greater the number of waves sampled in space-time, and unrealistically large amplitudes are likely to be sampled. Recently, Benetazzo et al.⁵⁹ proposed an approach to limit the effects of the unboundedness of rare Gaussian space-time events. In particular, they bound the range of wave crest amplitudes of a given sea state in the finite interval $[0, h_{max}]$ as waves inevitably break and limit crest heights. The threshold $h_{max} = \xi_{max} H_s$ is chosen based on wave observations. For Draupner, Andrea and El Faro sea states Benetazzo et al. ⁵⁹ set $\xi_{max} = 1.55$, but both Andrea and El Faro attained larger amplitudes (see Table 1). Such an approach was pioneered by Collins⁴⁵ and Battjes⁴⁶ in the early 70s (see Thornton and Guza⁴⁷ for a comprehensive review). Hereafter, we refer to it as the Collins-Battjes method. In simple words, it relies on defining a sea state of N waves whose sample space consists of NP_{max} non-breaking waves and $N(1-P_{max})$ breaking waves. Here, $P_{max} = Pr(h < h_{max})$ is the probability, or fraction of waves that do not break. All the mass probability of breaking waves is concentrated at $h = h_{max}$ and the associated probability density function (pdf) of crest heights has a Dirac delta spike $\delta(h - h_{max})$ of mass $(1 - P_{max})$ at h_{max} so that the probability of the entire sample space is 1. The pdfs reported by Benetazzo et al.⁵⁹ in their Eqs. (13) and (16) miss the Dirac delta spike to concentrate the probability mass $(1 - P_{max})$ at h_{max} . Following Collins-Battjes method⁴⁵⁻⁴⁷, we bound the Tayfun-Fedele exceedance probability in Eq. (4) as

$$P_{\text{ST,bound}}(\xi; A, D) = \begin{cases} P_{\text{ST}}(\xi; A, D), & \xi \le \xi_{max}, \\ 0, & \xi > \xi_{max}. \end{cases}$$
(8)

The correct pdf must have a Dirac delta spike at $\xi = \xi_{max}$ as

$$p_{\text{ST,bound}}(\xi; A, D) = \begin{cases} -\frac{dP_{\text{ST,bound}}}{d\xi}, & \xi \leq \xi_{max}, \\ (1 - P_{max})\delta(\xi - \xi_{max}), & \xi = \xi_{max}, \\ 0, & \xi > \xi_{max}, \end{cases}$$
(9)

where $P_{max} = Pr(h < h_{max}) = 1 - P_{\text{ST,bound}}(\xi_{max}; A, D)$ and the probability of the entire sample space adds up to 1. The space-time probability of exceedance can be estimated by the associated Gumbel model¹⁵

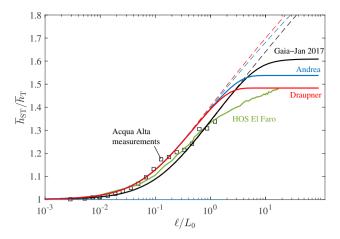


Figure 9. Space-time extremes: theoretical ratio $\overline{h}_{\rm ST}/\overline{h}_{\rm T}$ as a function of the area width ℓ/L_0 for El Faro, Draupner and Andrea rogue sea states as well as the sea states at the peaks of Gaia and Jan 2017 storms. $\overline{h}_{\rm ST}$ is the mean maximum surface height expected over the area ℓ^2 during a sea state of duration D=1 hours and $\overline{h}_{\rm T}$ is the mean maximum surface height expected at a point. For comparisons, the empirical ST ratio from the El Faro HOS simulations (green solid line) together with the experimental observations at the Acqua Alta tower (squares) are also shown 16 . L_0 is the mean wavelength. Dashed lines denote the unbounded Gaussian predictions, whereas solid lines denote the bounded predictions using Collins-Battjes method $^{45-47}$ with $\xi_{max}=1.55$.

$$P_{Gumbel}(\xi) = 1 - \exp\left[-\exp\left(-\beta(\xi - \xi_{\rm m})\right], \qquad 0 \le \xi \le \xi_{max},\tag{10}$$

limited in the range $[0, \xi_{max}]$. The expected maximum surface height follows by integrating the exceedance probability over the finite range

$$\xi_{\rm ST} = \overline{h}_{\rm ST}/H_{\rm s} = \int_0^{\xi_{max}} P_{\rm ST,bound}(\xi; A, D) \, d\xi. \tag{11}$$

The nonlinear mean maximum surface or crest height $h_{\rm T}$ expected at a point during the time interval D follows from Eq. (3) by setting $M_2=M_3=0$ and $M_1=N_{\rm D}$. We note in passing that for Gaia and Jan 2017 storms the unbounded TF model well describes the statistics of large crests at a point (see Figs. 6 and 7), and so for El-Faro¹³, Andrea and Draupner¹² rogue waves. The second-order form of the ST model above ($\Lambda=0$) has been implemented in WAVEWATCH III⁶⁰. The linear limit follows from Eq. (3) by setting $\mu=0$ and $\Lambda=0$.

To characterize rare space-time occurrences in third-order nonlinear random seas we consider the threshold $h_q = \xi_q H_s$ exceeded with probability q by the maximum surface height $\eta_{\rm max}$ over an area A during a sea state of duration D. This satisfies

$$P_{\text{ST},bound}(\xi_q; A, D) = q. \tag{12}$$

In simple words, the maximum surface height $\eta_{\rm max}$ observed within the area A during D exceeds the threshold h_q only in qN realizations of an ensemble of N identical sea states. Consider the ST ratio $\overline{h}_{\rm ST}/\overline{h}_{\rm T}$, where $\overline{h}_{\rm ST}$ is the mean maximum surface height expected over the area ℓ^2 during a sea state of duration D=1 hour and $\overline{h}_{\rm T}$ is the mean maximum surface height expected at a point during the same time interval D. Figure 9 displays the unbounded ST ratios (dashed lines) as a function of the area width ℓ/L_0 for the El Faro, Draupner and Andrea sea states 13 as well as the sea states at the peaks of Gaia and Jan 2017 storms. We also depict the bounded ratios (solid lines) using Collins-Battjes method and the threshold $\xi_{max}=1.55$ as in Benetazzo et al. 59. The ST ratios for Draupner and Andrea are estimated using the European Reanalysis (ERA)-interim data 52. For comparisons, we also report the empirical ST ratio from the HOS-simulated El Faro sea state 13 and the experimental observations at the Acqua Alta tower 16. Fedele et al. 13 noted that the unbounded predictions fairly agree with the El Faro HOS simulations and observations for small areas ($\ell \leq L_0$), but they overestimate over larger areas. Note that Gaia and Jan 2017 give similar predictions and slightly smaller than the other sea states. As expected, the bounded predictions limit the growth of the statistical ensemble of waves in space-time. In particular, the maximum surface height expected over an area is bounded by the threshold $h_{max}=1.55H_{\rm S}$, and it does not exceed 1.6 times the surface height expected at a point.

Prediction of potential hazards for vessel navigation. Consider a vessel that navigates through a wave field of surface elevations $\eta(x, y, t)$ at a constant speed V along a straight path at an angle β with respect to the x axis of a fixed Cartesian frame (x, y, t) over a time interval D. Define the occurrence frequency, or exceedance probability $P_{\varepsilon}(h)$ that a rogue wave exceeding a crest height h is encountered by the vessel during its navigation path. $P_{\varepsilon}(h)$ can be estimated as the probability that the maximum surface height exceeds h over

the space-time volume V_e spanned by the vessel in motion. To do so, we draw on Fedele et al.¹³ and consider the encountered wave field η_c as seen by an observer in the frame (x_e, y_e, t) moving with the vessel, where

$$x = x_e + V\cos(\beta)t, \quad y = y_e + V\sin(\beta)t, \tag{13}$$

and (x, y, t) is the fixed Cartesian frame. The wave field encountered by the cruising vessel, or equivalently the field of surface fluctuations measured from the moving frame (x_e, y_e, t) is

$$\eta_c(x_e, y_e, t) = \eta(x_e + V\cos(\beta)t, y_e + V\sin(\beta)t, t). \tag{14}$$

For an observer in the fixed frame (x, y, t), η is a random wave field homogeneous in space and stationary in time, and so is the encountered field η_c as seen from an observer on the vessel in the moving frame (x_e, y_e, t) . However, η_c will be non-stationary for a generic navigation route and variable vessel's speed. The associated space-time covariance is given by

$$\Psi_c(X, Y, T) = \frac{1}{\eta_c(x_e, y_e, t)\eta_c(x_e + X, y_e + Y, t + T)} = \int S(f, \theta) \cos(k_x X + k_y Y - 2\pi f_e T) df d\theta, \quad (15)$$

where $S(f,\theta)$ is the directional wave spectrum of the sea state, $k_x = k\cos\theta$ and $k_y = k\sin\theta$, where k is the wavenumber related to the frequency f via the linear dispersion relation. The Doppler-shifted encountered, or apparent frequency is $^{61-63}$

$$f_{e} = f - kV \cos(\theta - \beta)/(2\pi). \tag{16}$$

When the vessel moves faster than the waves coming from a direction θ , the apparent frequency $f_e < 0$ and waves appear to move away from an observer on the vessel. In this case, the direction of those waves should be reversed⁶¹, i.e. $\theta = \theta + \pi$, and f_e set as positive. The spectral moments $m_{ijk}^{(e)}$ of the encountered random field η_c are given by 13

$$m_{ijk}^{e} = \frac{\partial^{i+j+k} \Psi_{c}}{\partial X^{i} \partial Y^{j} \partial T^{k}} \Big|_{X=Y=T=0} = \int S(f,\theta) k_{x}^{i} k_{y}^{j} f_{e}^{k} df d\theta.$$
 (17)

In the moving frame (x_e, y_e, t) , the vessel is stationary. As a consequence, the vessel spans the same area equal to its size $L \times W = 0$ over the duration D, i.e. it covers the space-time volume $V_e = LW = 0$ min. Here, L and W are the length and width of the vessel, respectively. The probability $P_e(h)$ that the maximum surface height over V_e exceeds the threshold h can be estimated from the spectral moments m_{ijk}^e of the encountered wave field using the space-time model^{13, 15, 52} in Eq. (3). In our analysis, we consider the worst-case scenario of the vessel cruising against the dominant wave direction of the sea state. In such conditions the moving vessel encounters more waves than if it were anchored at a fixed point because the apparent frequency of the encountered waves increases.

As an application, we hypothesize the scenarios of two vessels of the Israeli Navy fleet navigating at their cruise speeds against the dominant waves of the peak sea state of Gaia over a time interval of 10 min. We consider the Super Dvora Mk III-class patrol boat, a vessel of small size (length 20 m and width 6 m) cruising at the average speed of 50 km/h (27 knots) and maximum speed of 93 km/h (50 knots). We also consider the Sa'ar 6-class corvette, a large vessel (length 90 m and width 14 m) cruising at the average speed of 30 km/h (17 knots) and maximum speed of 50 km/h (27 knots). Figure 10 depicts the bounded predictions (solid lines) of the nonlinear threshold h_n/H_s exceeded with probability 1/n by the maximum surface height seen by the observer on the two vessels over the duration D = 10 min. We used the Collins-Battjes method 45-47 described in the preceding section and adopt $\xi_{max} = 1.68$, which is the crest height of the largest simulated wave of the El Faro sea state¹³. The unbounded Gaussian predictions are also reported (dashed lines). Consider an observer on the large vessel navigating against the dominant waves of the peak sea state of Gaia at the average cruise speed of 30 km/h. The observer sees the vessel as stationary and the spanned space-time volume is $V_e = 90 \times 14 \text{ m}^2 \times 10 \text{ min}$. The probability that the maximum surface height exceeds $h=1.6H_s\approx 9$ m (here, $H_s=6.16$ m, see Table 1) when the vessel cruises along the dominant direction of waves is close to $P_e = 1/1260$, as shown in Fig. 10. An observer on the small vessel navigating against the dominant waves at the average cruise speed of 50 km/h has a smaller probability $P_e = 1/4000$ to encounter the rogue wave, because the vessel's size is smaller than that of the large vessel. If the two vessels navigate along with the dominant waves P_e reduces to 1/2400 (large vessel) and 1/10,500 (small vessel), because they will encounter less waves as the apparent wave frequency reduces.

For an observer fixed at a point of the sea, P_e reduces to $1/10^5$ (see Fig. 8). We note in passing that if the two vessels navigate at their maximum speeds against the dominant waves, P_e increases to 1/3160 (small vessel) and 1/1000 (large vessel).

Figure 10 also depicts the predictions for the El Faro vessel during Hurricane Joaquin based on HOS simulations¹³. The data suggests that the El Faro vessel was drifting at an average speed of approximately 2.5 m/s prior to its sinking¹³. Over a time interval of D=10 min prior sinking the probability to encounter a rogue wave that exceeds the threshold $h=1.6H_s\approx14$ m is $P_e\approx1/400$, in agreement with the predictions by Fedele et al.¹³. Thus, we find the same probabilities for the hypothetical scenario of the El Faro vessel drifting through the sea state at the storm peak of Gaia. However, in this case the rogue wave would exceed a smaller height, i.e. $1.6H_s\approx9$ m since the significant wave height at the peak of Gaia ($H_s=6.16$ m) is smaller than that of the El Faro sea state ($H_s=9$ m, see Table 1).

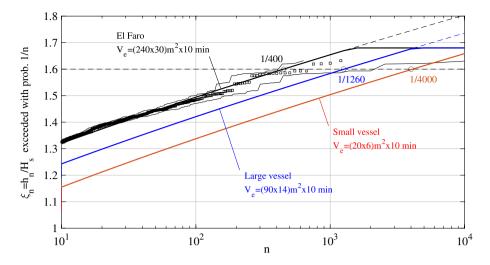


Figure 10. Theoretical bounded predictions (Collins-Battjes method⁴⁵⁻⁴⁷, $\xi_{max} = 1.68$) for the maximum surface height h_n/H_s exceeded with probability 1/n over the space-time volume $V_e = LW$ m² x D min spanned by the El Faro vessel (black lines), the Super Dvora Mk III-class patrol boat (red lines, small vessel) and the Sa'ar 6-class corvette (blue lines, large vessel). The vessels navigate against the dominant waves in the peak sea state of storm Gaia for a duration D. L and W are the length and width of the vessel. The numerical HOS predictions (squares) for El Faro are also reported¹³. Dashed lines denote the unbounded Gaussian predictions. Dashed horizontal line denotes the $1.6H_s$ threshold.

Conclusions

The Eastern Mediterranean is a basin of increasing interest being a climatology hot-spot and a natural treasures area, both ecologically and economically¹. However, there is a lack of studies on the metocean characteristics of its extreme sea states and large waves. Such studies are of paramount importance, especially in light of global warming where weather events are likely to become more severe and more hazardous to offshore and coastal structures as well as to vessel navigation.

To fill the gap of knowledge on metocean properties of the Eastern Mediterrenean sea storms, we have presented a scientific investigation and prediction of wave extremes of that area. Our statistical analysis indicates that the largest observed waves during the two major storms recorded in the last five years have characteristics quite similar to those displayed by the El Faro, Andrea, and Draupner rogue waves^{12, 13}. In particular, linear dispersive focusing enhanced by second order bound nonlinearities are the dominant factor in the generation of large waves. Third-order resonances and NLS-type modulational instabilities are ineffective in shaping large waves^{12, 19}. Finally, we have carried out a novel space-time analysis of the most intense sea states and demonstrated the ability to predict potential rogue hazards for ships of various sizes and cruise speed navigating within the rough waters of the analyzed storms.

Methods

Buoy measurements. The '3 metre' buoy manufactured by AXYS Technologies is located on the south part of the Exclusive Economic Zone of Israel. The buoy is moored at a depth of 260 m at the coordinates $(31^{o} 45' 41'' \text{ Lat.}, 34^{o} 20' 30'' \text{ Long.})$. This buoy operates by recording 18 minuets of heave displacements at the beginning of each hour with sampling frequency of $f_s = 7.14 \, \text{Hz.}$. Additional outputs are also elaborated by the internal buoy's software, i.e. the directional energy density spectrum spectrum $S(\omega, \theta)$ with a resolution of 3° covering 360°, and typical wave parameters such as the significant wave height, wave period and steepness. The same wave parameters were estimated independently from the the time series of the surface wave fluctuations provided by the buoy, and they were practically identical to the buoy's outputs. For a cross-site validation, the buoy's wave parameters have been compared to data acquired by the nearby onshore (i) Ashkelon port meteorological station and (ii) the DATAWELL Wave-Rider MKIII buoy's 40-km away from our buoy, and moored 2 km offshore at a 24-m depth. In both cases, the buoy's wave parameters showed similar trends to those acquired by the two instruments, while differing slightly in values. This is expected in light of the distance between the instruments' locations and the physical characteristics of shallow waters and onshore environments.

Wave parameters. The significant wave height H_s is defined as the mean value $H_{1/3}$ of the highest one-third of wave heights. It can be estimated either from a zero-crossing analysis or more easily but approximately from the wave omnidirectional spectrum $S_o(f) = \int_0^{2\pi} S(f,\theta) d\theta$ as $H_s \approx 4\sigma$, where $\sigma = \sqrt{m_0}$ is the standard deviation of surface elevations, $m_j = \int S_o(f) f^j df$ are spectral moments. Further, $S(f,\theta)$ is the directional wave spectrum with θ as the direction of waves at frequency f, and the cyclic frequency is $\omega = 2\pi f$. In this paper, we use the spectral-based estimate, which according to our wave measurements is 5% - 10% larger than $H_{1/3}$ estimated from the measured time series.

The dominant wave period $T_p=2\pi/\omega_p$ refers to the cyclic frequency ω_p of the spectral peak. The mean zero-crossing wave period T_0 is equal to $2\pi/\omega_0$, with $\omega_0=\sqrt{m_2/m_0}$. The associated wavelength $L_0=2\pi/k_0$ follows from the linear dispersion relation $\omega_0=\sqrt{gk_0\tanh(k_0d)}$, with d the water depth. The mean spectral frequency is defined as $\omega_m=m_1/m_0^{23}$ and the associated mean period T_m is equal to $2\pi/\omega_m$. A characteristic wave steepness is defined as $\mu_m=k_m\sigma$, where k_m is the wavenumber corresponding to the mean spectral frequency ω_m^{23} . The following quantitites are also introduced: $q_m=k_md$, $Q_m=\tanh q_m$, the phase velocity $c_m=\omega_m/k_m$, the group velocity $c_\sigma=c_m\left[1+2q_m/\sinh(2q_m)\right]/2$.

velocity $c_g = c_m \left[1 + 2q_m/\sinh(2q_m)\right]/2$. The spectral bandwidth $\nu = (m_0 m_2/m_1^2 - 1)^{1/2}$ gives a measure of the frequency spreading. The angular spreading $\sigma_\theta = \sqrt{\int_0^{2\pi} D(\theta)(\theta - \theta_m)^2 d\theta}$, where $D(\theta) = \int_0^\infty S(\omega,\theta) d\omega/\sigma^2$ and $\theta_m = \int_0^{2\pi} D(\theta)\theta d\theta$ is the mean direction. Note that $\omega_0 = \omega_m \sqrt{1 + \nu^2}$. An alternative measure of spectral bandwidth is given by the Boccotti parameter $\psi^* = \psi(\tau^*)$, which is the absolute value of the first minimum of the normalized covariance function $\psi(\tau) = \frac{1}{\eta(t)\eta(t+\tau)}/\sigma^2$ of the zero-mean surface displacement $\eta(t)$, which is attained at $\tau = \tau^{*66}$ and $\ddot{\psi}^* = \ddot{\psi}(\tau^*)$ the corresponding second derivative with respect to τ^{66} .

The wave skewness λ_3 and the excess kurtosis λ_{40} of the zero-mean surface elevation $\eta(t)$ are given by

$$\lambda_3 = \overline{\eta^3}/\sigma^3, \qquad \lambda_{40} = \overline{\eta^4}/\sigma^4 - 3. \tag{18}$$

Here, overbars imply statistical averages and σ is the standard deviation of surface wave elevations. Note that λ_3 was first derived by ⁶⁷. For second-order waves in deep water ²⁵

$$\lambda_3 \approx 3\mu_m (1 - \nu + \nu^2),\tag{19}$$

and18

$$3\mu_m(1 - \sqrt{2}\nu + \nu^2) < \lambda_3 < 3\mu_m. \tag{20}$$

Here, ν is the spectral bandwidth defined above and the characteristic wave steepness $\mu_m=k_m\sigma$, where k_m is the wavenumber corresponding to the mean spectral frequency ω_m^{23} . For narrow-band waves, ν tends to zero and the associated skewness $\lambda_{3,NB}=3\mu_m^{23-25}$. The excess kurtosis of weakly nonlinear random seas

$$\lambda_{40} = \lambda_{40}^d + \lambda_{40}^b \tag{21}$$

comprises a dynamic component λ_{40}^d due to nonlinear quasi-resonant wave-wave interactions ^{17, 26} and a Stokes bound harmonic contribution λ_{40}^b . In deep water it reduces to the simple form $\lambda_{40,NB}^b = 18 \mu_m^2 = 2 \lambda_{3,NB}^2$ where $\lambda_{3,NB}$ is the skewness of narrowband waves²³. In deep water, the dynamic component λ_{40}^d is given in terms of a six-fold integral ^{17, 19} that depends on the Benjamin-Feir index $BFI = \sqrt{2} \mu_m / \nu$ and the parameter $R = \sigma_\theta^2 / 2 \nu^2$, which is a dimensionless measure of the multidirectionality of dominant waves^{19, 26, 69}.

The Tayfun-Fedele model for crest heights. The probability $P(\xi)$ that a wave crest observed at a fixed point of the ocean in time exceeds the threshold ξH_s can be described by the third-order Tayfun-Fedele model²⁴,

$$P_{TF}(\xi) = \Pr[h > \xi H_s] = \exp(-8\xi_0^2) \left[1 + \Lambda \xi_0^2 \left(4\xi_0^2 - 1\right)\right],\tag{22}$$

where ξ_0 follows from the quadratic equation $\xi = \xi_0 + 2\mu \, \xi_0^{223}$. Here, the Tayfun wave steepness $\mu = \lambda_3/3$ is of $O(\mu_m)$ and it is a measure of second-order bound nonlinearities as it relates to the skewness λ_3 of surface elevations²⁵. The parameter $\Lambda = \lambda_{40} + 2\lambda_{22} + \lambda_{04}$ is a measure of third-order nonlinearities and is a function of the fourth order cumulants λ_{nm} of the water surface elevation η and its Hilbert transform $\hat{\eta}^{24}$. In particular, $\lambda_{22} = \frac{\eta^2 \hat{\eta}^2}{\eta^2} / \sigma^4 - 1$ and $\lambda_{04} = \frac{\hat{\eta}^4}{\eta^4} / \sigma^4 - 3$. In our studies Λ is approximated solely in terms of the excess kurtosis as $\Lambda_{appr} = 8\lambda_{40}/3$ by assuming the relations between cumulants $\lambda_{04} = \lambda_{40}/3$ and $\lambda_{04} = \lambda_{40}/3$. These, to date, have been proven to hold for linear and second-order narrowband waves only²⁷. For third-order nonlinear seas, our numerical studies indicate that $\Lambda \approx \Lambda_{appr}$ within a 3% relative error in agreement with observations λ_{10} .

our numerical studies indicate that $\Lambda \approx \Lambda_{\rm appr}$ within a 3% relative error in agreement with observations^{71,72}. For second-order seas, referred to as Tayfun sea states⁷³, $\Lambda = 0$ only and P_{TF} in Eq. (22) yields the Tayfun (T) distribution²³

$$P_T(\xi) = \exp(-8\xi_0^2). \tag{23}$$

For Gaussian seas, $\mu=0$ and $\Lambda=0$ and P_{TF} reduces to the Rayleigh (R) distribution

$$P_R(\xi) = \exp(-8\xi^2).$$
 (24)

Fedele and Tayfun²⁵ proved that the Tayfun distribution represents an exact result for large second order wave crest heights and it depends solely on the steepness parameter defined as $\mu = \lambda_3/3$.

The modified narrowband (MNB) model. Tayfun and Alkhalidi⁴¹ introduced the MNB model as an improvement of the conventional weakly-nonlinear second-order finite-depth narrowband model so as to avoid certain unrealistic features of that model in relatively shallow waters and to extend its applicability to highly nonlinear waves propagating from deep water to the shoaling and surf zones. The MNB exceedance probability for crest heights is given by

$$P_{MNB}(\xi) = \Pr[h > \xi H_s] = \exp(-8\xi_0^2),$$
 (25)

where ξ_0 follows from the quadratic equation 11, 41

$$\xi = \left(\xi_0 + 2\varepsilon \xi_0^2\right)/\alpha_1,$$

where

$$\varepsilon = 0.3571\lambda_3 - 0.0227\lambda_3^2 + 0.0444\lambda_3^3,$$

$$\alpha_1 = 1 + 0.0146\lambda_3 + 0.0147\lambda_3^2 + 0.0219\lambda_3^3,$$

and the associated excess kurtosis

$$\lambda_{40,MNB} = 0.0276\lambda_3 + 1.4275\lambda_3^2 - 0.0063\lambda_3^3$$

The MNB model is valid over the range of skewness values $0 \le \lambda_3 \le 2$.

The Forristall model. The exceedance probability is given by⁴³

$$P_F(\xi) = \exp(-(\xi/\alpha)^{\beta}), \tag{26}$$

where $\alpha = 0.3536 + 0.2561S_1 + 0.0800U_r$, $\beta = 2 - 1.7912S_1 - 0.5302U_r + 0.284U_r^2$ for multi-directional (short-crested) seas. Here, $S_1 = 2\pi H_s/(gT_m^2)$ is a characteristic wave steepness and the Ursell number $U_r = H_s/(k_m^2d^3)$, where k_m is the wavenumber associated with the mean period $T_m = m_0/m_1$ and d is the water depth.

The generalized Boccotti and Tayfun models for crest-to-trough (wave) heights. The third-order nonlinear statistics for crest-to-trough wave heights is described in terms of the generalized Boccotti distribution^{42,66}.

$$P_B(y) = \Pr[H > y H_s] = \frac{1 + \ddot{\psi}^*}{\sqrt{2 \ddot{\psi}^* (1 + \psi^*)}} \exp\left(-\frac{4 y^2}{1 + \psi^*}\right) \left[1 + \frac{\Lambda y^2}{1 + \psi^*} \left(\frac{y^2}{1 + \psi^*} - \frac{1}{2}\right)\right], \quad (27)$$

where the Boccotti parameter ψ^* and $\ddot{\psi}^*$ are defined above in the section where statistical parameters are described. For Gaussian seas ($\Lambda=0$), the original Boccotti model⁶⁶ is recovered

$$P_B(y) = \Pr[H > y H_s] = \frac{1 + \ddot{\psi}^*}{\sqrt{2 \, \ddot{\psi}^* (1 + \psi^*)}} \exp\left(-\frac{4 \, y^2}{1 + \psi^*}\right),\tag{28}$$

The Tayfun model for wave heights is given by^{24, 44}

$$P_T(y) = \Pr[H > y H_s] = \sqrt{\frac{1 + r_m}{2r_m}} \left(1 + \frac{1 - r_m^2}{64r_m y^2} \right) \exp\left(-\frac{4y^2}{1 + r_m} \right), \tag{29}$$

where $r_m = r(T_m/2)$ is the value of the envelope r(t) of the covariance $\psi(t)$ at $t = T_m/2$.

Space-time statistical parameters. For space-time extremes, the coefficients in Eq. (3) are given by 15,48

$$\begin{split} M_3 = & 2\pi \frac{D}{T} \frac{\ell_x}{L_x} \frac{\ell_y}{L_y} \alpha_{xyt}, \\ M_2 = & \sqrt{2\pi} \left(\frac{D}{T} \frac{\ell_x}{L_x} \sqrt{1 - \alpha_{xt}^2} + \frac{D}{T} \frac{\ell_y}{L_y} \sqrt{1 - \alpha_{yt}^2} + \frac{\ell_x}{L_x} \frac{\ell_y}{L_y} \sqrt{1 - \alpha_{xy}^2} \right), \\ M_1 = & N_D + N_x + N_y, \end{split}$$

where

$$N_D = \frac{D}{\overline{T}}, \qquad N_x = \frac{\ell_x}{\overline{L_x}}, \qquad N_y = \frac{\ell_y}{\overline{L_y}}$$

are the average number of waves occurring during the time interval D and along the x and y sides of length ℓ_x and ℓ_y respectively. They all depend on the mean period \overline{T} , mean wavelengths $\overline{L_x}$ and $\overline{L_y}$ in x and y directions:

$$\overline{T} = 2\pi \sqrt{\frac{m_{000}}{m_{002}}}, \qquad \overline{L_x} = 2\pi \sqrt{\frac{m_{000}}{m_{200}}}, \qquad \overline{L_y} = 2\pi \sqrt{\frac{m_{000}}{m_{020}}}$$

and

$$\alpha_{xyt} = \sqrt{1 - \alpha_{xt}^2 - \alpha_{yt}^2 - \alpha_{xy}^2 + 2\alpha_{xt}\alpha_{yt}\alpha_{xy}}.$$

Here,

$$m_{ijk} = \iint k_x^i k_y^j f^k S(f, \theta) df d\theta$$

are the moments of the directional spectrum $S(f, \theta)$ and

$$\alpha_{xt} = \frac{m_{101}}{\sqrt{m_{200}m_{002}}}, \qquad \alpha_{yt} = \frac{m_{011}}{\sqrt{m_{020}m_{002}}}, \qquad \alpha_{xy} = \frac{m_{110}}{\sqrt{m_{200}m_{020}}}.$$

Data availability

All the publicly available data for El-Faro are posted on the National Transportation Safety Board (NTSB) website (https://www.ntsb.gov).

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Author contributions

The concept and design was provided by F.F. and D.L. F.F. coordinated the scientific effort. The formal data analysis was performed by S.K. The space-time wave analysis was performed by F.F. The overall supervision was provided by D.L. and F.F. All authors made incisive intellectual contributions, participated in the analysis and interpretation of results and the writing of the manuscript.

Competing interests

The authors declare no competing interests.

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