

When Consumers Learn, Money Burns:
Signaling Quality Via Advertising
With Observational Learning And Word Of Mouth

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Abstract

This research analyzes a firm's investment in advertising that signals quality when consumers learn about quality not only from such advertising but also from interactions with other consumers in the form of observational learning or word of mouth. Further, word of mouth interactions may involve under-reporting (not everyone shares experiences), positivity (positive experiences are communicated more widely than negative ones) or negativity (negative experiences are communicated more widely than positive ones). The analysis focuses on whether a firm should advertise more or less aggressively in the presence of such consumer interactions as compared to their absence, and offers four key insights. First, consumer interactions can amplify the signaling effect of advertising, and as a consequence, to prevent mimicking it may be optimal for a high quality firm to become more aggressive and spend more on advertising to signal quality in the presence of such interactions, than without. Second, as under-reporting increases, it can be optimal to reduce advertising, sometimes significantly. Third, with increasing positivity, it can be optimal to increase advertising; and finally, even with increasing negativity, under certain conditions it may still be optimal to increase advertising rather than decreasing it.

Keywords: experience goods, advertising, observational learning, word of mouth, positivity, negativity, game theory, signaling, behavioral industrial organization.

1 Introduction

Every year firms introduce new products and services in many categories. While consumers are often aware of the existence of upcoming new goods, a priori they may be uncertain about their quality and fit with their needs. This is especially the case for experience goods, where uncertainty may be resolved post consumption, but consumption requires purchase. To persuade consumers to buy their experience goods, firms often use costly signals of quality. Advertising is an example of such a signal. These signals, when credible, help consumers update their beliefs about a firm's quality.

Past research has extensively analyzed the strategic signaling role that advertising plays in shaping consumer expectations. Since the early works of Nelson, it has been well recognized that advertising can serve as a credible signal of quality for experience goods (e.g., Nelson 1970, 1974, 1978; Schmalensee 1978; Kihlstrom and Riordan 1984; Milgrom and Roberts 1986; Hertzendorf 1993; Zhao 2000; Orzach et al. 2002). Even if the firm does not communicate any specific information about its product or service attributes in its advertising, a high amount of investment in advertising can be sufficient to rationally persuade a consumer that only a firm that believes it would be able to recoup this investment (e.g., via repeat sales) would choose to make it. This “burning money” narrative (e.g., Nelson (1974); Milgrom and Roberts (1986); Hertzendorf (1993); Zhao (2000)) suggests that “*purely dissipative¹ costly advertising can serve as a credible signal of quality for experience goods.*”

Firm communications (e.g., advertising or other promotional money burning initiatives), however, are not the only means by which consumers learn about quality. Consumers also rely on other information sources, most notably observed purchase decisions and opinions of peers (e.g., Godes and Mayzlin 2004; Zhang 2010; Godes 2012; Kuksov et al. 2013). Surveys show that consumers could be 4-5 times more likely to rely on friends and family than TV/print/social media (Bjornland et al. 2015) when it comes to learning information about new products and services. Ongoing developments in information technologies and social

¹Advertising that does not directly impact demand: i.e., $\frac{\partial(\text{demand})}{\partial(\text{advertising})} = 0$.

media make it easier to observe, record, store and share consumer choices and peer opinions. Consequently, such media are playing a more prominent role in the consumer decision process of trying and adopting new products. This is consistent with the emergence of companies such as *yelp.com* and the competitive advantage enjoyed by *amazon.com* through its product reviews system. Consumer reliance on peers can change the incentives for firms to rely on advertising as the means of signaling quality. This paper considers how a firm should adjust its advertising signaling efforts when consumers also rely on observations or word of mouth to learn about product quality.

In practice, information sharing among consumers is characterized by a few common patterns. First, not all consumers share their opinions with their peers. For example, consumers may intentionally or un-intentionally withhold information. This may happen due to several reasons, including the fact that sharing information involves effort (e.g., Hu et al. 2009). It is not unusual to observe that a small proportion of customers buying a particular product choose to write a review for it. For instance, Anderson and Simester (2014) in the analysis of their data found that “approximately 1.5% of the firm’s customers” write reviews. Further, such under-reporting behavior may be accentuated for certain types of experiences: for example, in certain cases, negative experiences may be more likely to be shared than positive ones. For instance, Dixon et al. (2010) found from customer surveys that while a quarter of consumers who had a positive experience told ten or more people about it, about half the consumers who had a negative experience told ten or more people about it. Such behavior is consistent with prospect theory which suggests that losses have a stronger impact than gains (Kahneman and Tversky 1979). Similar behavioral patterns might also arise, for example, when considering attributes that consumers typically regard as minimum requirements, such as airline punctuality (Brandt 1988), where consumers may primarily discuss negative experiences (e.g., “my flight left one hour late”), rather than positive ones (e.g., “the flight departed on time”). In contrast, in other situations, a buyer might prefer to abstain from complaining to peers about a recent purchase, for example, consistent with an

impression management motive (Berger 2014). This suggests that while in some contexts, consumers might be more likely to encounter negative opinions, in others consumers might be more likely to encounter positive ones (e.g., Chevalier and Mayzlin 2006; Berger and Milkman 2012).

Given that information sharing among consumers may not always provide an unbiased reflection of true quality, a firm faces the critical challenge of optimally determining its communication strategy when consumer beliefs are shaped by such interactions. Analyzing this firm decision is the central research goal of this paper. Specifically, this research investigates how distinct interactions among consumers affect the equilibrium advertising spending of a firm seeking to credibly signal quality. It is imperative to note at the outset that besides advertising, other firm decisions such as price can also play an important role in signaling quality (along with distribution, sales, etc). However, to focus attention on studying the effects of consumer interactions on advertising, throughout this paper, the analysis assumes an exogenous price that does not depend on quality.²

The first key insight from this research is that compared to their absence, the presence of information sharing mechanisms such as observational learning and word of mouth can make a firm more aggressive in its advertising spending. Second, when some consumers do not report their experiences, advertising can be lower compared to the case where all consumers report experiences. Under certain conditions, this spending can be even lower than that in the absence of any communication among consumers. Third, as positivity in word of mouth increases (i.e., consumers are more likely to share positive experiences relative to negative ones), it can be optimal for a firm to respond aggressively by increasing its advertising spending. Finally, as negativity in word of mouth increases (i.e., consumers are more likely to share negative experiences relative to positive ones), under certain conditions, it may still be optimal for a firm to respond aggressively by increasing its advertising spending, rather than decreasing it.

²For a model of price as a signal of quality in the presence of word of mouth, please see Guadalupi (2016).

The rest of this paper is structured as follows. Section 2 reviews the past research relevant to our problem of study. Section 3 analyzes a simple model for the firm's advertising signaling decision without any consumer interactions. Doing so has two benefits. First, it introduces the reader to the modeling framework used subsequently throughout the paper. Second, it establishes a baseline for a firm's equilibrium advertising spending in the absence of consumer interactions, which serves as a meaningful comparison for the equilibrium advertising levels derived in each of the subsequent models. This model is extended in Section 4 to allow for observational learning. Section 5 models information sharing among consumers via various forms of word of mouth communication. In Section 6, the effects of under-reporting, negativity, and positivity in word of mouth communication on advertising are analyzed. Section 7 concludes by discussing the main findings and ideas for future research.

2 Literature Review

This research analyzes the impact of information sharing among consumers via observational learning or word of mouth on advertising, where advertising signals the quality of an experience good to consumers. To place this research in the appropriate context, the discussion below is organized into two parts. The first part discusses past theoretical findings regarding the role of advertising as well as other marketing actions in signaling quality to consumers. The second part discusses existing work regarding how observational learning and word of mouth may influence consumers' beliefs regarding quality.

Early research on advertising as a mechanism for communicating quality argued that the amount spent on advertising, irrespective of its content, could potentially convey useful information regarding quality (Nelson 1970, 1974, 1978). Subsequent work by Kihlstrom and Riordan (1984) formalized these arguments by showing that the existence of market mechanisms such as fixed and variable production costs can lead to a positive relationship between advertising and quality.

Milgrom and Roberts (1986) extended these arguments by studying a monopolist using dissipative advertising along with an introductory price to signal the quality of its experience good to consumers who repeatedly purchase the good. They showed that in equilibrium, a firm may use both price and advertising to credibly signal its quality. The use of two decisions, both advertising and price, allowed a firm to be more efficient in its signaling; consequently, relying on only one decision to signal quality could lead to inefficiencies in terms of costs incurred by a firm to credibly signal quality. Further exploring efficiency, Hertzendorf (1993) extended the work of Milgrom and Roberts (1986) to situations where advertising may be noisy, and showed that advertising takes place only when price is not correlated with quality. Zhao (2000) considered the informative role of advertising along with its role as a quality signal. Given its informative role, advertising was no longer purely dissipative; consequently in equilibrium Zhao (2000) argued that one could observe a negative correlation between advertising and quality.

Past research has also shown that a firm may signal quality to consumers not only through advertising and price, but also through many other specific marketing actions. For instance, a firm may signal quality via the reputation of its retail partner (Chu and Chu 1994); a money back guarantee (Moorthy and Srinivasan 1995); the duration of its warranty (Stock and Balachander 2005); perceived scarcity (Balachander 2001); specialization (Kalra and Li 2008) or the set of features offered (Bhardwaj et al. 2008). In addition to specific marketing actions, previous market shares have also been shown to serve as signals of product quality (Caminal and Vives 1996). Exploring the interaction between demand and marketing actions, Miklos-Thal and Zhang (2013) analyzed the signaling implications of a firm's marketing activities where the role of marketing is to expand demand for search goods. They showed that under certain conditions toning down marketing efforts can be a credible signal of quality. This is because when the early demand is small relative to later demand, later consumers are likely to attribute low sales in the early market to insufficient marketing, thus enhancing their perceptions of firm quality. Thus, the review above highlights the strategic importance

placed by firms on one or more marketing decisions as effective and efficient means of credibly communicating quality to consumers. In this research, the focus will be on one firm decision, advertising, as the mechanism for signaling quality.

Apart from firm communication, past research has also looked at how consumers may obtain information on their own to learn about quality. Mayzlin and Shin (2011) have examined the role of search and advertising content, and showed that uninformative advertising along with search can serve as a credible signal of quality. Besides conducting their own search, consumers routinely learn from their peers, e.g., via observational learning or word of mouth. For instance, Zhang (2010) studied sequential choices made by kidney transplant candidates on a waiting list and found that observed prior choices are used to infer product quality. Banerjee and Fudenberg (2004) analyzed successive generations of consumers choosing between two options and rationally relying on word of mouth from a sample of prior consumers to infer option utility. They also studied biases in word of mouth, such as over-reporting certain types of experiences; however, their focus was on identifying conditions that lead to consumers selecting the efficient option. While this current research also considers biased word of mouth behaviors, the focus here is instead on how these behaviors affect the advertising spending of a firm seeking to credibly signal its quality.

Past research has also explored the impact of the informational content of word of mouth on advertising. Chen and Xie (2005) studied the effect of product review content on a firm's advertising, and showed that product reviews that include a recommendation are complementary to advertising, but product reviews that contain a description are not. In a subsequent study, Chen and Xie (2008) found that word of mouth and informative advertising (e.g., about product attributes) might be complements or substitutes depending on product costs and the level of consumer expertise. Mayzlin (2006) studied consumers learning online from anonymous recommendations sent to them either by the firm or by consumers, and found that despite promotional chat from the firm, word of mouth still remained a persuasive communication device for consumers. Godes (2012) analyzed the impact of reference pro-

grams in business-to-business markets, and showed that when a firm's information is noisy, offering peer references can serve as a credible signal of quality. More recently, in an empirical analysis of reviews in the hotels industry, Hollenbeck et al. (2019) found that reviews and advertising are substitutes, as hotels with higher ratings spend less on advertising. In contrast to much of this previous research, this current paper focuses on dissipative (i.e., non-informative) advertising, and finds that word of mouth and advertising can often be complements instead of substitutes, when the role of advertising is to signal quality.

Finally, word of mouth communication can also impact the price set by a firm to signal quality. Guadalupi (2016) studied the impact of word of mouth on prices for a new experience good in a dynamic model and found that a high quality firm uses a low introductory price to signal its quality, as experimentation with price is costly and hence can only be afforded by the high type. In this paper, rather than focusing on how word of mouth affects price, the focus is instead on how word of mouth changes the amount spent on dissipative advertising by a high quality firm to credibly signal its type.

Thus, a review of the past literature indicates that while there exists research analyzing the role of marketing actions such as advertising on credibly signaling firm quality, and research analyzing consumer interactions in inferring quality, research regarding the effects of information sharing mechanisms among consumers on the firm's use of advertising for credibly signaling quality is lacking. Understanding these effects, and studying how a firm should approach its advertising decision in the presence of information sharing behaviors to credibly signal quality is hence the central focus of this paper.

3 A Baseline Model For Advertising

The baseline model is motivated by the classic signaling model of Milgrom and Roberts (1986). Consider a monopolist who has developed a new experience good of quality q_0 . Quality is defined as an exogenously specified *ex ante* probability with which consumers

expect the consumption experience to be positive; i.e., $P(\text{positive experience}|\text{consumption}) = q_0$ and thus $P(\text{negative experience}|\text{consumption}) = 1 - q_0$. In other words, quality reflects the likelihood of “fit” between the firm’s good and a consumer’s needs. Given its experiential nature, whether or not a consumer likes the product is revealed to her only post consumption, and is independent of the experiences of other consumers. *Ex ante*, let product quality be high (H) with probability κ and low (L) with probability $1 - \kappa$, and $0 < L < H < 1$.

Let the market consist of consumers who belong to one of two segments: influentials or followers (as is common in the diffusion and social influence literature, e.g., Katz and Lazarsfeld 1955, Coleman et al. 1957, Rogers 1962, Watts and Dodds 2007, Van den Bulte and Joshi 2007, Miklos-Thal and Zhang 2013). Let the size of the influential segment be normalized to 1, and let $s > 0$ denote the relative size of the follower segment.

Prior to consumption, a consumer’s (expected) utility is specified to have three components, $v + E[q_0] - p$. The first component v is a baseline utility that captures benefits from consumption in this product category. Assume that consumers are heterogeneous in terms of this baseline utility v , such that v is uniformly distributed over $[0, 1]$ within each consumer segment. The second component is expected utility that is dependent on the quality of the experience. Assume that consumers value positive experiences more than negative experiences. Let consumer utility from a positive experience be set to 1 and that from a negative experience be set to 0. Thus, if expected product quality is $E[q_0]$, this second component of expected utility becomes $(E[q_0])(1) + (1 - E[q_0])(0) = E[q_0]$. The third component of consumer utility is an exogenously specified fixed price p paid by consumers in order to purchase this good. All consumers have available to them an outside option whose utility is normalized to 0. Consumers purchase the firm’s good if their expected utility from its consumption is higher than that for the outside good. The post-purchase consumption experience reveals whether or not the product fits the needs of an individual consumer. This fit is consumer-specific and remains unchanged for future purchase and consumption occasions.

The game consists of two periods (the sequence of events in this baseline model is sum-

marized in Figure 1). Prior to the start of the game, all players are assumed to be aware of the existence of the new product and its *ex ante* quality probability distribution, but not the realized quality type. At the beginning of the first period, nature selects the type and only the firm learns its product quality. Consumers are unaware of the realized type, and there is no credible direct alternative way via which the firm can provide this information to consumers except via its introductory advertising spending. Thus, advertising spending serves as the mechanism available to the firm to signal its quality to consumers.

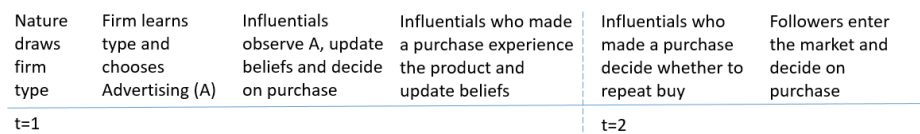


Figure 1: Timing of the baseline game.

Influentials enter the market earlier, in period 1, and are targeted by the firm via advertising. Followers enter the market later, in period 2, and in some cases may learn from influentials either through observations or word of mouth communication (Ellison and Fudenberg 1995, Dellarocas 2003, Kuksov and Xie 2010, Zhang 2010). Assume that followers do not observe the advertising signal.³ The assumption that only some of the consumers (in this case, the influentials) observe the firm’s advertising or promotional efforts is consistent with several industry practices such as limited product launches, the premiere of a movie, press releases and publications of new product advertisements in specialized magazines and websites. Many of these initiatives do not reach all consumers in the market, but instead are mainly targeted (directly or indirectly) towards early adopters. In addition, for various reasons, typically for every advertising campaign there always are some consumers who are not reached by the campaign.

The choice of advertising as the firm’s sole decision variable for signaling quality also merits additional discussion. First, as discussed in Section 2, firms have at their disposal numerous marketing decisions such as price, promotions, the choice of distribution channels

³This assumption will be relaxed subsequently to analyze the effects of signaling to followers.

and product warranties as mechanisms for signaling quality. Second, past research shows that the use of two signals primarily allows a firm to be more efficient in credibly communicating quality, as compared to relying on just one signal. For instance, Milgrom and Roberts 1986 analyze the role of both price and advertising as signals of a firm's quality, and show that there are efficiency gains when using multiple signals of quality. Third, there are numerous contexts where certain marketing decisions such as prices may not serve as an effective signal of quality. For example, in many industries such as movies, music, online services, games and sporting events, prices may be set based on external factors that do not vary as a function of product quality. In such settings, advertising may still serve as a credible device for signaling quality. Consequently, to focus attention on the strategic role of advertising as a signaling device in the presence of consumer interactions, in this paper, assume that the firm does not rely on any other marketing actions (e.g., price, promotions and distribution channels) for signaling its quality to consumers. Consistent with past literature in this context (e.g., Schmalensee 1978; Mayzlin 2006; Moorthy 2012; Kuksov et al. 2013), assume that price p is exogenous and fixed across quality types.

In this baseline model, there is no interaction between influentials and followers. Therefore, in the second period, followers decide between consuming either the firm's good or the outside good based solely on their prior beliefs. The derivation of consumer demand and firm profit depends on the specific value of the exogenous fixed price. In the analysis that follows, for ease of exposition, let price vary between $H < p \leq 1$. For $1 < p < 1 + L$, the analysis follows analogous steps and leads to qualitatively similar outcomes in most cases.⁴ For simplicity, assume that there are no marginal costs of production.

The solution concept for this game is perfect Bayesian equilibrium (PBE), where the firm chooses its advertising spending consistent with the equilibrium beliefs of all consumers, and beliefs of all consumers are consistent with the firm's equilibrium signaling strategy.

⁴When the outcomes are different, those are stated in the corresponding sections. Prices outside the $(H, 1 + L)$ range are not considered in this analysis, since they lead to uninteresting outcomes: e.g., all influentials always buy when $p \leq H$ and q_0 is believed to be H , and none of the influentials buy when $p \geq 1 + L$ and q_0 is believed to be L .

A pooling equilibrium exists when either firm, regardless of its quality type, spends the same amount on advertising; and a separating equilibrium exists when advertising spending differs across the two firm types. Signaling games such as the above commonly suffer from the existence of multiple pooling and separating equilibria. The analysis that follows first characterizes the existence of pooling equilibria, followed by that of separating equilibria. The analysis of pooling equilibria reveals the following:⁵

Lemma 1. *Any pooling equilibrium that exists for the baseline signaling game does not survive the Intuitive Criterion (Cho and Kreps 1987) refinement.*

The proof for Lemma 1 (as well as for all subsequent lemmas) is provided in the Appendix. The intuition behind the result in Lemma 1 is that the H type is always willing to deviate from a pooling equilibrium by spending a sufficiently high amount on advertising that only it can afford. This is because the H type delivers more positive experiences to influentials than the L type, and hence is able to recoup its investment in advertising in the later period via higher repeat sales. Using analogous arguments relying on the Intuitive Criterion (Cho and Kreps 1987), pooling equilibria are ruled out for all models in the rest of this paper. Consequently, for the remainder of the paper, the discussion focuses on characterizing separating equilibria.

In any potential separating equilibrium for this baseline game, let A_i denote the advertising spending by type i , for $i \in \{L, H\}$. Upon observing the firm's advertising decision, on the equilibrium path, influentials update their beliefs as follows: $P(q_0 = H|A_H) = 1$ and $P(q_0 = L|A_L) = 1$. An influential aware of the firm's type knows the *ex ante* probability q_0 of her experience being positive. Hence, equilibrium demand in the first period among influentials is $1 + q_0 - p$.⁶

⁵The initial results presented here for the baseline signaling game are conceptually well known in the signaling literature. They are presented here in order to mathematically characterize the specific outcomes for the current modeling setup, and to aid comparison with subsequent models developed in this paper.

⁶In a separating equilibrium, for influentials, $E[q_0] = q_0$. Thus, an influential would buy if $v + q_0 - p > 0$. Thus, demand from influentials in period 1 is $\int_{p-q_0}^1 dv = 1 + q_0 - p$.

Influentials have an opportunity to repeat their purchase in the second period. Influentials with a high baseline utility (i.e., those with $v > p$) will repeat their purchase in period 2, regardless of the quality of their consumption experience (i.e., $\min_q v + q - p = v - p > 0$, when $v > p$).⁷ The remaining $(1 - p + q_0) - (1 - p) = q_0$ influentials who made a purchase in period 1 will repeat their purchase in period 2 only if they had a positive experience, an event which happens with probability q_0 . Hence, repeat sales are $1 - p + H^2$.

Followers enter the market in the second period. Given no interactions with the influentials, their demand is driven solely by their priors. Hence, analogous to the derivation for influential demand in period 1, follower demand in period 2 is $(1 + (\kappa H + (1 - \kappa)L) - p)s$.

Thus, profit for the high type can now be written as:

$$\pi_H = p(1 + H - p) + p(1 - p + H^2) + p(1 + (\kappa H + (1 - \kappa)L) - p)s - A_H;$$

whereas profit for the low type can be written as:

$$\pi_L = p(1 + L - p) + p(1 - p + L^2) + p(1 + (\kappa H + (1 - \kappa)L) - p)s - A_L.$$

For a separating equilibrium to exist, neither type should have an incentive to mimic the other. Let $\pi_{q_0q'_0}$ denote the profit for type q_0 when mimicking the other type q'_0 :

$$\pi_{HL} = p(1 + L - p) + p(1 - p + LH) + p(1 + (\kappa H + (1 - \kappa)L) - p)s - A_L,$$

$$\pi_{LH} = p(1 + H - p) + p(1 - p + HL) + p(1 + (\kappa H + (1 - \kappa)L) - p)s - A_H;$$

The following lemma characterizes the existence of separating equilibria for this game:

Lemma 2. *There exist separating equilibria for the baseline signaling game that survive the Intuitive Criterion (Cho and Kreps 1987) refinement, and in any such equilibria, $A_L^* = 0$ and $A_H^* = p(H - L)(1 + L)$.*

Lemma 2 confirms the established wisdom (e.g., Schmalensee 1978; Kihlstrom and Riordan 1984; Milgrom and Roberts 1986; Bagwell 2007) that purely dissipative advertising can serve as a credible signal of product quality. Specifically, the optimal amount spent on advertising by the high type in a separating equilibrium is just enough to prevent mimicking

⁷When $1 < p < 1 + L$, no such influentials exist.

by the low type. In this baseline model, if a low type mimics the actions of the high type, it gains an incremental demand of $(H - L)$ among the influential segment in period 1 due to influentials now believing that the type is high. A proportion of these additional influentials gain a positive experience, which then translates to an incremental demand of $(H - L)L$ due to repeat sales from them in period 2. Followers in period 2 are unaffected by the low type's mimicking actions, hence do not contribute to any change in sales. Put together, the high type must spend $p(H - L)(1 + L)$ to prevent the low type from mimicking its actions, and finds it profitable to do so.

As expected, equilibrium advertising by the high type is increasing in p as well as H , and is decreasing in L . This is because each of these changes – i.e., either an increase in p or H , or a decrease in L – makes it more attractive for a low type to mimic the high type. Advertising does not depend on prior beliefs about the firm's quality (κ) or the relative size of follower segment (s); since both these parameters only affect follower demand, and in this baseline model mimicking does not impact follower demand.

To assess the rationale behind the firm signaling only to influentials, consider now the potential outcomes in a model where the firm's signal can instead be observed by both influentials and followers. Analogous to the earlier analysis for Lemma 1, in this model as well no pooling equilibria survive the Intuitive Criterion (Cho and Kreps 1987). In any potential separating equilibria, demand from influentials also remains the same as in the previous model. But demand from the followers is no longer the same. Followers enter in period 2 and now observe the firm's advertising, update their beliefs and thus behave similar to the influentials in period 1. Hence, profit for the high type in this case can be written as:

$$\pi_H = p(1 + H - p) + p(1 - p + H^2) + p(1 + H - p)s - A_H;$$

whereas profit for the low type can be written as:

$$\pi_L = p(1 + L - p) + p(1 - p + L^2) + p(1 + L - p)s - A_L.$$

The following lemma characterizes the existence of separating equilibria for this game:

Lemma 3. *For the signaling game when both segments observe the signal, there exist sepa-*

rating equilibria that survive the Intuitive Criterion (Cho and Kreps 1987) refinement, and in any such equilibria, $A_L^ = 0$ and $A_H^* = p(H - L)(1 + L + s)$.*

Comparing the equilibrium advertising spending by the H type from Lemma 2 and Lemma 3, note that advertising spending is higher when both segments observe the signal as compared to the case when only influentials observe the signal. This is because when both segments observe the signal, mimicking is more attractive to the low type; since mimicking increases demand not only among influentials but also among followers. As a consequence, in order to separate, the high type has to spend more on advertising to credibly signal its type. For this reason, equilibrium advertising when targeting both segments also increases with the relative size of the follower segment (s).

Interestingly, and more importantly, profits for both types are lower when a firm targets the signal to both influentials and followers, as compared to profits when targeting influentials only. For the H type, when targeting both segments, the optimal advertising spending is higher (increases by $p(H - L)s$), and profit from the followers is also higher (increases by $p(H - L)(1 - \kappa)s$). As is evident, the net effect on profit is negative for the H type. For the L type, when followers also observe the signal, its demand from followers drops by $\kappa(H - L)s$, harming its profits compared to the case where only influentials are targeted by the firm's signaling efforts. Thus, this comparison indicates that for this baseline model, a firm deciding which consumer segments to target with its signal would prefer to target influentials only.

Having established a baseline case, the rest of the paper now proposes and analyzes distinct information sharing mechanisms between influentials and followers, and compares how these affect the optimal advertising spending by the high type. In the next section, consider first the effects of observational learning, where followers can observe the decisions made by influentials before making their purchase decision.

4 Advertising With Observational Learning

A simple mechanism for consumers to learn about the quality of a product from other consumers is by observing the actions of those consumers. In the baseline framework presented in Section 3, observational learning can be modeled in terms of the volume as well as the specific decisions of influentials that are observed by followers.

Volume refers to the fraction of influentials that a follower is able to observe. In terms of volume, consider two cases: [a] each follower observes the purchase decision of a single influential; or [b] each follower observes the purchase decisions of all influentials. In terms of the specific decisions observed, consider three cases: [i] each follower observes influentials' period 1 purchase decision only; [ii] each follower observes influentials' period 2 repeat purchase decision only; or [iii] each follower observes influentials' purchase decisions in period 1 as well as repeat purchase decision in period 2.

The analysis that follows focuses on case [a] where a follower observes a single influential, while case [b] where a follower observes all influentials is analyzed in the Online Appendix⁸. Within each case, all three specific cases of observed decisions are analyzed.

The baseline framework presented in Section 3 is now modified to permit a follower to observe the decisions of a single influential. To do so, let each follower be randomly paired with one influential, thus allowing the follower to observe the initial and/or repeat purchase decisions of this influential, depending on the specific case being analyzed. In each case, in period 2, a follower learns from the observed influential behavior by updating her beliefs about the firm type per Bayes' rule, and decides between consuming either the firm's good or the outside good. In any potential separating equilibrium, total first period purchases from influentials are $(1 + q_0 - p)$ and total second period repeat purchases are $(1 - p + q_0^2)$.

First consider case (i), where each follower observes an influential's initial purchase decision in period 1. Let $E[q_0]_p$ denote a follower's expected quality after observing an influential making a purchase, and $E[q_0]_{np}$ denote a follower's expected quality upon observing no pur-

⁸For more details, please refer to the Online Appendix A.

chase. The following lemma describes the ordering of follower beliefs.

Lemma 4. *Follower expected quality upon observing an influential purchase in period 1 is higher than prior expected quality which is higher than expected quality upon observing an influential not purchase in period 1 (i.e., $E[q_0]_p > \kappa H + (1 - \kappa)L > E[q_0]_{np}$).*

The order of follower beliefs in Lemma 4 is as expected: compared to their prior, expected quality is higher upon observing an influential purchase, since purchase is more likely when the type is high. Along the same lines, compared to their prior, expected quality is lower upon observing no purchase, since no purchase is more likely when the type is low.

In any potential separating equilibrium, profit for the high type can be written as:

$$\pi_H = p(1 + H - p) + p(1 - p + H^2) + p((1 + H - p)(1 + E[q_0]_p - p) + (p - H)(1 + E[q_0]_{np} - p))s - A_H;$$

whereas profit for the low type can be written as:

$$\pi_L = p(1 + L - p) + p(1 - p + L^2) + p((1 + L - p)(1 + E[q_0]_p - p) + (p - L)(1 + E[q_0]_{np} - p))s - A_L.$$

The following lemma characterizes the existence of separating equilibria for this game:

Lemma 5. *For the signaling game when a follower observes an influential's first period purchase decision, there exist separating equilibria that survive the Intuitive Criterion (Cho and Kreps 1987) refinement, and in any such equilibria, $A_L^* = 0$ and $A_H^* = p(H - L)(1 + L + (E[q_0]_p - E[q_0]_{np})s)$.*

The intuition behind the result in Lemma 5 is analogous to that of the result in Lemma 2: optimal advertising for a high type in a separating equilibrium is an amount that is just enough to prevent mimicking by the low type. Were the low type to mimic the high type, in addition to the incremental demand from influentials, demand also changes among followers. This is because an incremental fraction $(H - L)$ of followers observe an influential having made a purchase rather than a non-purchase. These followers update their beliefs to $E[q_0]_p$

instead of the less favorable beliefs $E[q_0]_{np}$. A comparison of the equilibrium advertising spending in the presence of observational learning (Lemma 5) versus its absence (Lemma 2) leads to the following proposition:

Proposition 1. *Advertising spending by the high type in a separating equilibrium satisfying the Intuitive Criterion (Cho and Kreps 1987) refinement is higher in the game with followers observing influentials' initial purchases than in the game with no consumer interactions.*

Popular press and lay beliefs often portray advertising and consumer learning as substitutes. One reason for this is the argument that both may convey information about firm quality. Hence intuitively one might expect that in the presence of consumer learning, e.g. via observations, a high type might have to spend less on advertising compared to the case where consumer learning does not take place. Proposition 1 argues that this may not always be the case. When a follower observes an influential's purchase decision, she makes inferences regarding the plausible firm type. A low type can potentially take advantage of such learning, which can be misguided if the influentials were to be fooled into believing that the firm is of the high type. Consequently, this leads to separation being costlier for a high type firm, as compared to the situation where no observational learning takes place. Thus, a high type may indeed need to optimally spend more on advertising in the presence observational learning by consumers, than without.

While the discussion so far focused on case (i) where a follower observes an influential's initial purchase in period 1, now consider case (ii) where each follower is randomly matched with an influential and observes this influential's repeat purchase decision in period 2, prior to making their own purchase decision. The analysis proceeds along similar lines as before. Let $E[q_0]_r$ denote a follower's expected quality after observing an influential making a repeat purchase, and $E[q_0]_{nr}$ denote a follower's expected quality upon observing no repeat purchase. The following lemma describes the ordering of follower beliefs:

Lemma 6. *Follower expected quality upon observing an influential repeat purchase in period 2 is higher than prior expected utility which is higher than expected utility upon observing an influential not purchase in period 2 (i.e., $E[q_0]_r > \kappa H + (1 - \kappa)L > E[q_0]_{nr}$).*

The order of follower beliefs in Lemma 6 is also as expected: compared to their prior, expected quality is higher upon observing an influential repeat purchase, since repeat purchase is more likely when the type is high. Along the same lines, compared to their prior, expected quality is lower upon observing no repeat purchase, since no repeat purchase is more likely if when the type is low.

In any potential separating equilibrium, profit for the high type can be written as:

$$\pi_H = p(1 + H - p) + p(1 - p + H^2) + p((1 - p + H^2)(1 + E[q_0]_r - p) + (p - H^2)(1 + E[q_0]_{nr} - p))s - A_H;$$

whereas profit for the low type can be written as:

$$\pi_L = p(1 + L - p) + p(1 - p + L^2) + p((1 - p + L^2)(1 + E[q_0]_r - p) + (p - L^2)(1 + E[q_0]_{nr} - p))s - A_L.$$

The following lemma characterizes the existence of separating equilibria for this game:

Lemma 7. *For the signaling game when a follower observes an influential's second period repeat purchase decision, there exist separating equilibria that survive the Intuitive Criterion (Cho and Kreps 1987) refinement, and in any such equilibria, $A_L^* = 0$ and $A_H^* = p(H - L)(1 + L + L(E[q_0]_r - E[q_0]_{nr})s)$.*

The intuition behind the result in Lemma 7 is analogous to that of the result in Lemma 5, except that here followers update their beliefs to $E[q_0]_r$ and $E[q_0]_{nr}$. A comparison of the equilibrium advertising spending in the presence of observational learning (Lemma 7) versus its absence (Lemma 2) leads to the following proposition:

Proposition 2. *Advertising spending by the high type in a separating equilibrium satisfying the Intuitive Criterion (Cho and Kreps 1987) refinement is higher in the game with followers observing influentials' repeat purchases than in the game with no consumer interactions.*

Finally, following analogous arguments, it can be shown that for case (iii) – where each follower is randomly matched with an influential and observes this influential’s initial purchase decision in period 1 as well as her repeat purchase decision in period 2, prior to making their own purchase decision – advertising spending by the high type in a separating equilibrium is higher than that in the game with no consumer interactions.⁹

Having analyzed the effects of observational learning on a firm’s advertising strategy, the next section now focuses on the effects of word of mouth, where followers can learn from influentials about their consumption experiences before making a purchase decision.

5 Advertising With Word of Mouth

A second mechanism for consumers to learn about the quality of a product from other consumers is by talking with those consumers, i.e., via word of mouth. In the baseline framework presented in Section 3, learning via word of mouth can also be modeled in terms of the volume as well as the specific information shared between influentials and followers.

Here, volume refers to the fraction of influentials that a follower talks with. In terms of volume, again consider two cases: [a] each follower talks with a single influential; or [b] each follower talks with all influentials. In terms of the specific information shared, consider three cases: [i] influentials share the quality of their experience (i.e. whether their experience was positive or negative; if they have no experience to share, they do not share anything); [ii] influentials share their ex-post consumption surplus (this shared surplus reflects how satisfied an influential is with her purchase decision, given her baseline utility); or [iii] some influentials share their ex-post consumption surplus while others share their observed advertising signal.

The analysis that follows focuses on case [a] where each follower talks with a single influential. Section 5.1 analyzes case [i] where the influential shares the quality of her experience, if any. Section 5.2 analyzes case [ii] where the influential shares her ex-post consumption

⁹When $1 < p < 1 + L$, some additional conditions need to be imposed. For the details of this analysis, please refer to Online Appendix B.

surplus. The analysis for case [iii] where some influentials share their surplus whereas others share their observed advertising signal as well as for case [b] where each follower talks with all influentials under all three specific information sharing mechanisms ([i]-[iii]) is presented in the Online Appendix¹⁰.

5.1 Sharing Experiences Via Word of Mouth

Consider first the case where each follower is randomly paired with one influential, and this influential shares the quality of her experience, if any, with the follower. Specifically, information shared could be one of three types: [i] the influential shares a positive experience; [ii] the influential shares a negative experience; or [iii] the influential did not have any experience to share and hence stays silent. Here, silence is also informative and is correctly interpreted by the follower as the influential not making a purchase. Thus, a follower learns from the shared experience, updates her beliefs about the firm type per Bayes' rule, and decides whether to purchase the firm's good or the outside good.

Denote follower's expected quality after hearing a positive, negative or no experience by $E[q_0]_+$, $E[q_0]_-$ and $E[q_0]_s$, respectively. In any potential separating equilibrium, period 1 demand from influentials is $(1 + q_0 - p)$. A fraction q_0 of those purchases lead to positive experiences, whereas a fraction $1 - q_0$ lead to negative experiences. Hence, per Bayes' rule:

$$\begin{aligned} E[q_0]_+ &= \frac{(1+H-p)H^2\kappa+(1+L-p)L^2(1-\kappa)}{(1+H-p)H\kappa+(1+L-p)L(1-\kappa)} \\ E[q_0]_- &= \frac{(1+H-p)(1-H)\kappa H+(1+L-p)(1-L)(1-\kappa)L}{(1+H-p)(1-H)\kappa+(1+L-p)(1-L)(1-\kappa)} \\ E[q_0]_s &= \frac{(p-H)\kappa H+(p-L)(1-\kappa)L}{(p-H)\kappa+(p-L)(1-\kappa)} \end{aligned}$$

Since a high type is more likely to provide positive experiences, $E[q_0]_+$ is higher than the prior expected quality $\kappa H + (1 - \kappa)L$. However, $E[q_0]_-$ can be higher or lower than the prior, depending on values of L and H . Finally, $E[q_0]_s$ is lower than the prior, since an influential is more likely to buy when the type is high in which case silence is less likely to

¹⁰For more details, please refer to the Online Appendices C "Sharing Surplus Or Signal Via Word Of Mouth When A Follower Talks With An Influential" and D "Word Of Mouth When Followers Talk With All Influentials"

be encountered. The following lemma describes the ordering of these follower beliefs.

Lemma 8. *For $H < p \leq 1$, follower expected quality upon hearing a positive experience is higher than expected quality upon hearing a negative experience which is higher than expected quality upon hearing no experience/silence (i.e., $E[q_0]_+ > E[q_0]_- > E[q_0]_s$).*

Compared to positive word of mouth, beliefs under silence are penalized. This is because silence communicates that the influential did not buy, a more likely choice when the type is low. Under a negative experience, beliefs are penalized as well because a negative experience is more likely when type is low. However, beliefs are also rewarded because any experience implies that an influential chose to purchase, which is more likely when the type is high. Thus, a follower has a higher expected quality after hearing negative word of mouth as compared to encountering silence.¹¹

In any potential separating equilibria, profit for the high type can be written as:

$$\pi_H = p(1 + H - p) + p(1 - p + H^2) + p((1 + H - p)(H(1 + E[q_0]_+ - p) + (1 - H)(1 + E[q_0]_- - p)) + (p - H)(1 + E[q_0]_s - p))s - A_H;$$

whereas profit for the low type can be written as:

$$\pi_L = p(1 + L - p) + p(1 - p + L^2) + p((1 + L - p)(L(1 + E[q_0]_+ - p) + (1 - L)(1 + E[q_0]_- - p)) + (p - L)(1 + E[q_0]_s - p))s - A_L.$$

The following lemma characterizes the existence of separating equilibria for this game:

Lemma 9. *For the signaling game with word of mouth sharing experiences, there exist separating equilibria that survive the Intuitive Criterion (Cho and Kreps 1987) refinement, and in any such equilibria, $A_L^* = 0$ and $A_H^* = p(H - L)(1 + L + s(L(E[q_0]_+ - E[q_0]_s) + (1 - L)(E[q_0]_- - E[q_0]_s)))$.*

¹¹For $p \in (1, 1 + L)$, the reverse can be true under certain conditions. Quality beliefs are penalized less after exposure to silence because purchases become less likely under higher prices. It can be shown that within this range of prices, when both L and H are sufficiently high (i.e., $p - \sqrt{p - 1} < H < 1$ and $(1 - p - Hp + p^2)/(p - H) < L < H$), silence leads to more favorable beliefs than negative word of mouth.

Thus, in this model, advertising is a credible signal of quality as in Milgrom and Roberts (1986), but the mechanism here is distinct. In Milgrom and Roberts (1986), advertising is a credible signal due to the existence of repeat sales. In this model above, advertising is a credible signal due to word of mouth between an influential and a follower, along with repeat sales. In fact, even in the absence of repeat sales, existence of word of mouth communication between an influential and a follower is enough for advertising to serve as a credible signal of quality. This is because the H type has more to gain than the L type from being perceived as a high quality as opposed to a low quality firm. Being perceived as a high quality firm helps reach more followers, and being the H type helps deliver more positive experiences and hence more positive word of mouth, than the L type. This allows the high type to spend more than what the low type would be able to afford in order to credibly signal its quality. Repeat sales are included in the model because they are required to achieve separation in the baseline model with no interactions between influentials and followers¹².

Interestingly, the equilibrium advertising in Lemma 9 can be rewritten as $A_H^* = p(H - L) + p(H - L)L + p(H - L)(L(E[q_0]_+ - E[q_0]_s) + (1 - L)(E[q_0]_- - E[q_0]_s))s$. Intuitively, this expression represents the total potential gain to a low type from mimicking the high type. The first two terms represent gains from the influential segment. Being perceived as a high instead of a low type increases first and second period influential demand and hence potential profit by $p(H - L)$ and $p(H - L)L$, respectively. The incremental influential demand in period 1 also has an impact on demand in the follower segment in period 2. Some followers, who in equilibrium would have been exposed to silence, now instead hear either positive (with probability L) or negative (with probability $1 - L$) word of mouth. This translates to a profit gain of $p(L(E[q_0]_+ - E[q_0]_s) + (1 - L)(E[q_0]_- - E[q_0]_s))s$ from the follower segment. From these gains, it is easy to see that the equilibrium advertising spending increases with the size of the follower segment (s).¹³ A comparison of the equilibrium advertising spending

¹²For the analysis of a model without repeat sales, please refer to the Online Appendix E “Advertising With Word Of Mouth And Without Repeat Sales”.

¹³The relationship between the equilibrium advertising spending and prior beliefs (κ) is not straightforward. A numerical analysis indicates that under distinct market conditions, advertising spending could

in the presence of word of mouth sharing experiences (Lemma 9) versus its absence (Lemma 2) along with the results of Lemma 8 leads to the following proposition:

Proposition 3. *Advertising spending by the high type in a separating equilibrium satisfying the Intuitive Criterion (Cho and Kreps 1987) refinement is higher in the game with word of mouth sharing experiences than in the game with no consumer interactions (for $H < p \leq 1$).*

Analogous to the discussion following Proposition 1, lay beliefs might lead one to expect that in the presence of word of mouth communication a high type should have to spend less on advertising. The above result shows that this need not be the case: since sharing experiences improves follower expected quality (see Lemma 8), this increases the incentives for a low type to mimic the actions of the high type, leading to the high type having to spend more on advertising in the presence of word of mouth.¹⁴

The analysis above assumes that in period 1, influentials decide whether or not to consume the good based on their expected utility from consumption during period 1. An interesting possibility to consider is that influentials could be forward looking in their behavior: i.e., they decide whether or not to consume the good based on their expected utility from consumption across both time periods. With forward looking influentials, the key result in Proposition 3 – i.e., advertising spending by the high type in a separating equilibrium is higher in the presence of word of mouth than without – under certain conditions, still continues to hold.¹⁵

If followers also observe the signal, following analogous steps it is easy to show that equilibrium advertising spending by the high type is $p(H - L)(1 + L + s)$. This amount is higher than the amount spent on advertising in the presence of word of mouth when only influentials observe the signal (Lemma 9). It is also higher than the amount spent on

increase or decrease as consumers have more favorable prior beliefs (i.e., κ higher).

¹⁴For $1 < p \leq 1 + L$, $E[q_0]_-$ is not always greater than $E[q_0]_s$. Therefore, potential mimicking gains to the low type from shifting followers from silence into negative word of mouth become weaker. Nevertheless, in this higher price range, it can be shown that as long as H is not too extreme (i.e., $H < 0.96$), advertising by the high type in a separating equilibrium is still greater in the game with word of mouth than that in the baseline game with no consumer interactions.

¹⁵For more details, please refer to the Online Appendix F “Advertising With Forward Looking Consumers”.

advertising when there is no word of mouth between influentials and followers (Lemma 2). Analogous to Section 3, in the presence of word of mouth, if the firm decides which consumer segments to target with its signal, it optimally chooses to signal to influentials only.¹⁶ This outcome – that it is optimal for the firm to target influentials only rather than both segments – continues to hold for all subsequent models analyzed in this paper.

5.2 Sharing Surplus Via Word of Mouth

Now consider the case where each follower is randomly paired with one influential, and this influential shares her surplus with the follower. Specifically, an influential shares whether she made a purchase or not, and if yes, the realized *ex-post* consumption surplus u . For an influential with baseline utility v , $u = v + q - p$, where $q \in \{0, 1\}$ is the realized quality of the consumption experience; which is positive ($q = 1$) with probability q_0 and negative ($q = 0$) otherwise. Interestingly, learning u is sufficient for a follower to infer with certainty the quality of an influential’s consumption experience (i.e., whether it was positive or negative). This is because $v \in (0, 1)$ with probability 1, and in this range *ex-post* surplus from a positive experience strictly dominates *ex-post* surplus from a negative experience ($\min_v(v + 1 - p) > 1 - p > \max_v(v + 0 - p)$). Thus, given u , $q = 1$ if $u > 1 - p$ and $q = 0$ if $u < 1 - p$. Given u and q , a follower can infer the influential’s baseline utility $v = u + p - q$, which is relevant information to a follower in updating her beliefs, as discussed next.

When randomly paired with an influential, a follower is likely to encounter either an influential who made a purchase, or one who did not make a purchase. First consider follower beliefs when randomly paired with an influential who made a purchase and shared her surplus, u . Given u , a follower infers the influential’s experience q and her baseline utility v , as described above; and subsequently updates her beliefs. The updating process differs based on whether the influential’s baseline utility is high or not, as follows.

Influentials with a sufficiently high baseline utility (i.e., $v > p - L$) always purchase in

¹⁶For more details, please refer to the Online Appendix G “Optimality Of Targeting Influentials Only.”

Influential's Baseline Utility	Influential's Shared Experience		
	Positive Experience	Negative Experience	No Purchase
$v \in (p - L, 1]$	$E[q_0]_{h+} = \frac{H^2\kappa + L^2(1-\kappa)}{H\kappa + L(1-\kappa)}$	$E[q_0]_{h-} = \frac{(1-H)\kappa H + (1-L)(1-\kappa)L}{(1-H)\kappa + (1-L)(1-\kappa)}$	-
$v \in (p - H, p - L)$	$E[q_0]_{mp} = H$		$E[q_0]_{nop} = \frac{(p-H)\kappa H + (p-L)(1-\kappa)L}{(p-H)\kappa + (p-L)(1-\kappa)}$
$v \in [0, p - H)$	-		

Table 1: Follower expected quality given influential's shared experience and baseline utility.

period 1, regardless of whether they believe the type is H or L . Hence their purchase decision in period 1 is not diagnostic about firm type. However, the quality of their experience q is informative. Let $E[q_0]_{h+}$ and $E[q_0]_{h-}$ denote follower expected quality after talking with a high baseline utility influential who had a positive and negative experience, respectively.

Accordingly, $E[q_0]_{h+} = \frac{H^2\kappa + L^2(1-\kappa)}{H\kappa + L(1-\kappa)}$ and $E[q_0]_{h-} = \frac{(1-H)\kappa H + (1-L)(1-\kappa)L}{(1-H)\kappa + (1-L)(1-\kappa)}$.

Now consider an influential with moderate baseline utility, i.e. $v \in (p - H, p - L)$. This influential decides to purchase in period 1 only if she believes the firm's type is H . Thus, if a follower is matched with an influential with moderate baseline utility who made a purchase, the follower infers that the firm's type must be H , regardless of the quality of the consumption experience. Let $E[q_0]_{mp}$ denote follower expected quality after talking with a moderate baseline utility influential who made a purchase. Accordingly, $E[q_0]_{mp} = H$.

Finally, consider a follower who is randomly matched with an influential who did not make a purchase, and hence has no consumption surplus to share. This is likely when the match is either with an influential with moderate baseline utility $v \in (p - H, p - L)$ who believes that the firm type is low; or with an influential with low baseline utility $v < p - H$, who does not purchase regardless of firm type. Let $E[q_0]_{nop}$ denote follower expected quality after talking with an influential who did not make a purchase. Accordingly, $E[q_0]_{nop} = \frac{(p-H)\kappa H + (p-L)(1-\kappa)L}{(p-H)\kappa + (p-L)(1-\kappa)}$. Table 1 summarizes follower expected quality under these various possible cases.¹⁷

¹⁷For completeness, note that there are two boundary cases, where $v = p - L$ and $v = p - H$, both of which have zero probability. When $v = p - L$, an influential is indifferent between purchasing or not if the firm type was believed to be L , whereas when $v = p - H$, an influential is indifferent between purchasing or not if the firm type was believed to be H .

In any potential separating equilibria, profits for the high type can be written as:

$$\pi_H = p((1 + H - p) + (1 - p) + H^2) + p((1 + L - p)(H(1 + E[q_0]_{h+} - p) + (1 - H)(1 + E[q_0]_{h-} - p)) + (H - L)(1 + H - p) + (p - H)(1 + E[q_0]_{nop} - p))s - A_H;$$

whereas profit for the low type can be written as:

$$\pi_L = p((1 + L - p) + (1 - p) + L^2) + p((1 + L - p)(L(1 + E[q_0]_{h+} - p) + (1 - L)(1 + E[q_0]_{h-} - p)) + (p - L)(1 + E[q_0]_{nop} - p))s - A_L.$$

The following lemma characterizes the existence of separating equilibria for this game:

Lemma 10. *For the signaling game with word of mouth sharing experiences, there exist separating equilibria that survive the Intuitive Criterion (Cho and Kreps 1987) refinement, and in any such equilibria, $A_L^* = 0$ and $A_H^* = p(H - L)(1 + L + (H - E[q_0]_{nop})s)$.*

The intuition behind this result is analogous to that discussed after Lemma 9. The equilibrium advertising spending by the high type corresponds to an amount that is just sufficient to prevent a low type from mimicking the high type. The gains from mimicking originate both from the influential and follower segment. The gains in the influential segment have the same structure as in previous models, i.e. improvement in first and second period revenues ($p(H - L)$ and $p(H - L)L$, respectively). The gains in the follower segment can be better understood by subsegmenting this group of consumers in terms of the baseline utility v of the influentials that they are matched with. On the high end of the baseline utility distribution, followers matched with influentials such that $v > p - L$ learn that these influentials either had a positive or a negative experience, because all of these influentials made a purchase. More importantly, the fraction of these influentials that had a positive experience depends on the true firm type q_0 and hence it is unaffected by mimicking efforts. On the low end of the baseline utility distribution, followers matched with influentials such that $v < p - H$ learn that these influentials did not purchase, which once again does not change regardless of the firm's mimicking efforts.

In the middle range, followers are matched with influentials such that $v \in (p - H, p - L)$.

In contrast with the other two cases, the behavior of this subsegment of influentials can indeed be affected by the mimicking efforts of the low type. This is because influentials in this subsegment purchase only if they believe the firm's type to be high. Therefore, if the low type spends $A_L^* = 0$, then these influentials believe the firm's type to be low and decide not to purchase. If instead, the low type mimics the high type by spending A_H^* , then these influentials are led to believe the firm's type is high, and hence purchase. Followers matched with these influentials then infer that the type must be high. Consequently, the mimicking gains in the follower segment are obtained among those who are matched with influentials with intermediate levels of baseline utility (i.e., a fraction $H - L$ of the followers) and for each of them expected quality becomes H instead of $E[q_0]_{nop}$. This yields gains in the follower segment equal to $p(H - L)(H - E[q_0]_{nop})s$.

A comparison of the equilibrium advertising spending in the presence of word of mouth sharing surplus (Lemma 10) versus its absence (Lemma 2) leads to the following proposition:

Proposition 4. *Advertising spending by the high type in a separating equilibrium satisfying the Intuitive Criterion (Cho and Kreps 1987) refinement is higher in the game with word of mouth sharing surplus than in the game with no consumer interactions.*

Having analyzed the effects of word of mouth on advertising, the next section discusses commonly observed word of mouth behaviors among consumers and models their implications on a firm's advertising strategy.

6 Effects Of Word of Mouth Behaviors

Consider the word of mouth model analyzed in Section 5.1, where a follower was randomly matched with an influential, and the influential shared her experience, if any, with the follower. In that model, it was assumed that an influential was willing to share information regarding her consumption when matched with a follower. However, as discussed in Section

2, in practice consumer behavior may differ. First, due to a variety of reasons, not all consumers may share their experiences, irrespective of whether the experience was positive or negative. This can lead to an under-reporting of experiences, which can be modeled as influentials sharing their experiences probabilistically. Next, when consumers share their experiences, the types of experiences that consumers share may vary in different contexts. For example, in some consumption situations, consumers might discuss negative experiences more than positive experiences. This can be modeled as the proportion of negative experiences shared being relatively higher than the proportion of positive experiences shared. Finally, at other times, consumers tend to discuss positive opinions more than negative ones (e.g., Chevalier and Mayzlin (2006); Berger and Milkman (2012)). This can be modeled as the proportion of positive experiences shared being relatively higher than the proportion of negative experiences shared. The analysis below presents three such models. For simplicity and to focus attention on the effects of word of mouth behaviors, let $\kappa = \frac{1}{2}$, $s = 1$ and $p = 1$.

6.1 Under-Reporting In Word of Mouth

To model under-reporting, let r denote the probability that an influential who gained an experience actually shares her experience via word of mouth when paired with a follower ($0 \leq r \leq 1$). At the extremes, when $r = 1$ word of mouth is unrestricted (as discussed in Section 5.1); whereas when $r = 0$ there is no word of mouth between influentials and followers (as discussed in Section 3). As r increases, the likelihood of reporting experiences increases, thus it follows that $1 - r$ denotes the extent of under-reporting of experiences. Followers are aware of r ; and account for it when updating their beliefs. The analysis of this model follows along the lines of that in Section 5.1, and leads to the following result:

Lemma 11. *In the presence of under-reporting in word of mouth, follower expected quality upon not hearing any experience from an influential ($E[q_0]_s = \frac{(1-Hr)H+(1-Lr)L}{1-Hr+1-Lr}$) is increasing in the extent of under-reporting ($1 - r$).*

It follows from Lemma 11 that follower expected quality under silence is higher in the presence of under-reporting in word of mouth ($r < 1$) as compared to that of unrestricted word of mouth ($\frac{(1-H)H+(1-L)L}{1-H+1-L}$). In other words, with under-reporting, follower beliefs are more “forgiving” under silence. With under-reporting, a follower knows that hearing no experiences is plausible not only when an influential decided not to make a purchase, but also when an influential made a purchase but did not share her experience. As under-reporting becomes more likely, the higher the chances that silence can be attributed to not sharing experiences. Further, there is more under-reporting when the firm is of a high type, since more influentials make a purchase when the type is high.

Follower beliefs upon hearing either a positive ($E[q_0]_+$) or a negative ($E[q_0]_-$) experience retain the same functional form as when $r = 1$. The following lemma characterizes the existence of separating equilibria for this game:

Lemma 12. *For the signaling game with under-reporting, there exist separating equilibria that survive the Intuitive Criterion (Cho and Kreps 1987) refinement, and in any such equilibria, $A_L^* = 0$ and $A_H^* = (H - L)(1 + L + r(L(E[q_0]_+ - E[q_0]_s) + (1 - L)(E[q_0]_- - E[q_0]_s)))$.*

A comparison of the equilibrium advertising spending in the presence of under-reporting (Lemma 12) versus unrestricted word of mouth (Lemma 9) lead to the following proposition:

Proposition 5. *Advertising spending by the high type in a separating equilibrium that survives the Intuitive Criterion (Cho and Kreps 1987) refinement is lower in the game with symmetric under-reporting than in the game with unrestricted word of mouth.*

Recall that the equilibrium advertising spending equals the potential gains to the low type from mimicking the high type, and these gains are a function of the increase in demand in the influential and follower segments. Compared to $r = 1$, when $r < 1$, gains in the follower segment are weaker for two reasons. First, the fraction of followers that hear a positive or negative experience instead of silence is smaller, by $1 - r$. Second, the improvement in

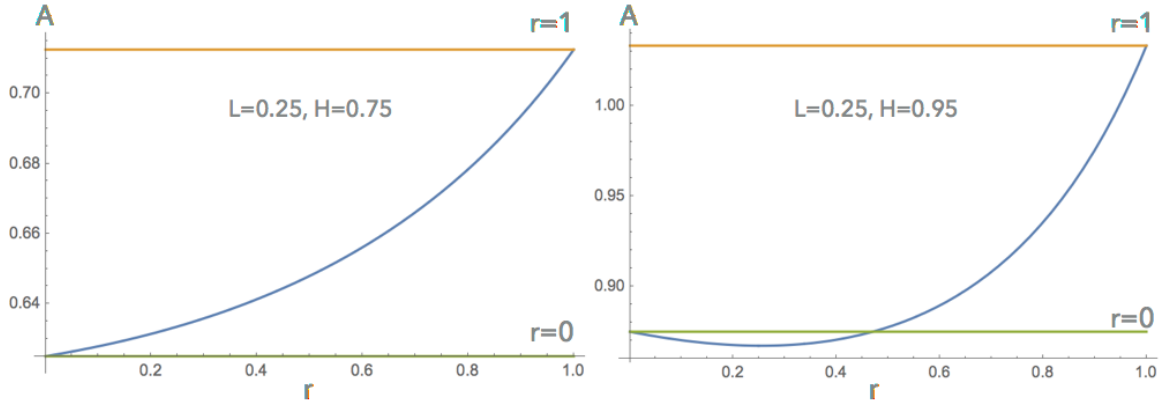


Figure 2: Equilibrium advertising spending as a function of word of mouth reporting.

expected quality beliefs (either $E[q_0]_+ - E[q_0]_s$ or $E[q_0]_- - E[q_0]_s$) is weaker, because per Lemma 11 beliefs when exposed to silence are more forgiving and favorable for $r < 1$.

Figure 2 illustrates an interesting relationship between equilibrium advertising spending by the high type and reporting levels of word of mouth for select values of H and L . In this figure, the bottom horizontal line represents advertising without any consumer interactions ($r = 0$) and the top horizontal line represents advertising with unrestricted word of mouth ($r = 1$). Per Proposition 5, with under-reporting in word of mouth, equilibrium advertising spending is always below the case of unrestricted word of mouth.

Interestingly, Figure 2 illustrates that equilibrium advertising spending is not always increasing in the level of reporting. In fact, there exist conditions under which advertising spending with under-reporting can be lower than that without word of mouth (as shown in the right panel). Recall that without word of mouth, when a low type mimics a high type, it gains profit among influentials but not among followers, since followers do not gain any additional information. With under-reporting in word of mouth, and particularly when reporting levels are low, follower quality beliefs are very forgiving about silence, such that their expected utility under silence is higher than that under negative word of mouth. Hence, when a low type mimics the high type, more influentials buy and talk to followers, but the net effect among followers is negative: while some followers are moved out of silence into hearing positive word of mouth, others are moved out of silence into hearing negative word

of mouth. Expected utility under negative word of mouth is lower than that under silence, leading to a reduction in the low type's gain among followers from mimicking. Consequently, equilibrium advertising by the high type falls below that of the case when there is no word of mouth among followers.

Having discussed the effects of symmetric under-reporting, the analysis below now discusses the effects of asymmetry in under-reporting on the firm's advertising strategy. For purposes of simplicity and to demonstrate the main insights, the analysis focuses on extreme cases of reporting asymmetry.

6.2 A Model Of Consumer Complaints

Consider now an extreme case where influentials do not share their experiences with followers if the experiences are positive, but may only share experiences when they are negative. Let r_n denote the probability that an influential shares a negative experience with a follower given that she actually had a negative experience. Thus, this model represents a market with negativity, where negativity increases with r_n . Similar to Section 6.1, equilibrium outcomes in this setting are compared to those where word of mouth is unrestricted.

Here, a follower encountering silence implies one of three possibilities: (i) the influential did not purchase; (ii) the influential had a positive experience and did not share it; or (iii) the influential had a negative experience and did not share it. As a consequence, follower expected quality under silence is $E[q_0]_s = \frac{(1-H(1-H)r_n)H+(1-L(1-L)r_n)L}{1-H(1-H)r_n+1-L(1-L)r_n}$. The following lemma compares follower beliefs under silence with and without negativity:

Lemma 13. *Follower expected quality on encountering silence in the presence of negativity $(\frac{(1-H(1-H)r_n)H+(1-L(1-L)r_n)L}{1-H(1-H)r_n+1-L(1-L)r_n})$ is higher than that under unrestricted word of mouth $(\frac{(1-H)H+(1-L)L}{1-H+1-L})$.*

Under unrestricted word of mouth, silence is only encountered when a follower interacts with an influential who did not make a purchase. With negativity, silence may also be

encountered in cases where an influential made a purchase but did not report the experience. Therefore, negativity makes a follower attribute silence not only to a no-purchase outcome, but also to the possibility that an influential purchased, had a positive experience but did not share it. This explains the result in Lemma 13 and why follower beliefs when exposed to silence are more favorable with negativity. The following lemma characterizes the existence of separating equilibria for this game:

Lemma 14. *For the signaling game with complaints, there exist separating equilibria that survive the Intuitive Criterion (Cho and Kreps 1987) refinement, and in any such equilibria, $A_L^* = 0$ and $A_H^* = (H - L)(1 + L + r_n(1 - L)(E[q_0]_- - E[q_0]_s))$.*

Comparing equilibrium advertising spending by the high type under negativity (Lemma 14) to that under unrestricted word of mouth (Lemma 9) leads to the following proposition:

Proposition 6. *Advertising spending by the high type in a separating equilibrium that survives the Intuitive Criterion (Cho and Kreps 1987) refinement is lower in the game with complaints than in the game with unrestricted word of mouth.*

To understand the intuition, first consider the special case of $r_n = 1$, i.e. all negative experiences but no positive experiences are shared. In this case, a follower who would have otherwise heard a positive experience if word of mouth were unrestricted would now be exposed to silence. Since follower expected utility is lower under silence than under hearing a positive experience, this weakens the gains for a low type from mimicking the high type. In a separating equilibrium, this leads to lower advertising spending by the high type.

Now let $r_n < 1$ so that not every negative experience is shared. In these cases, some followers who would have otherwise heard negative experiences are now exposed to silence. Under certain conditions (i.e., $L < H < 1 - L$), follower expected utility is lower under silence than under hearing a negative experience, and hence a similar logic holds. However, when these conditions do not hold, follower expected utility is higher under silence than

under hearing a negative experience. Even in such situations, mimicking gains from moving followers from hearing negative experiences into silence are not enough to match the loss from followers hearing positive experiences moving into silence, leading to lower equilibrium advertising for the high type.

To see how advertising changes with the extent of negativity in word of mouth (r_n), it is useful to first study the effect of negativity on follower expected quality under silence:

Lemma 15. *Follower expected quality under silence ($E[q_0]_s = \frac{(1-H(1-H)r_n)H+(1-L(1-L)r_n)L}{1-H(1-H)r_n+1-L(1-L)r_n}$) in the presence of negativity is decreasing in the level of negativity (r_n) when $L < H < 1 - L$ and increasing otherwise.*

When $H = 1 - L$, the probability of a follower being exposed to silence is equal under either type ($1 - L(1 - L)r_n = 1 - H(1 - H)r_n$). But when $H > 1 - L$, it is easy to verify that silence is more likely when the firm type is H . Moreover, the difference between these two silence probabilities becomes greater as negativity in word of mouth (r_n) increases. Consequently, an increase in negativity improves the chances that silence can be attributed to a high type, thus increasing follower expected quality under silence. On the other hand, when $H < 1 - L$, silence is more likely when the firm type is L . Consequently, follower expected quality under silence decreases with r_n .

Lemma 15 has important implications on how equilibrium advertising spending by the high type changes with negativity in word of mouth.

Proposition 7. *Advertising spending by the high type in a separating equilibrium that survives the Intuitive Criterion (Cho and Kreps 1987) refinement in the game with complaints is increasing in the extent of word of mouth negativity (r_n) iff $L < H < 1 - L$.*

As negativity increases, some followers who would have otherwise been exposed to silence are now exposed to complaints. When $L < H < 1 - L$, it is easy to verify that follower expected quality from hearing a complaint is higher than that under silence. This increases

the gains from mimicking, implying a greater advertising spending by the high type to credibly signal its type. A similar logic applies when $L < H < 1 - L$ does not hold.

6.3 A Model Of Consumer Compliments

Finally, consider the other extreme case where influentials do not share their experiences with followers if the experiences are negative, but only share experiences when they are positive. Let r_p denote the probability that an influential shares a positive experience with a follower given that she actually had a positive experience. Thus, this model represents a market with positivity, where positivity increases with r_p . The analysis here proceeds along the same lines as in Section 6.2. Here, follower expected quality under silence is $E[q_0]_s = \frac{(1-H^2r_p)H+(1-L^2r_p)L}{1-H^2r_p+1-L^2r_p}$. The following lemma characterizes the existence of separating equilibria for this game:

Lemma 16. *For the signaling game with compliments, there exist separating equilibria that survive the Intuitive Criterion (Cho and Kreps 1987) refinement, and in any such equilibria, $A_L^* = 0$ and $A_H^* = (H - L)(1 + L + r_p L(E[q_0]_+ - E[q_0]_s))$.*

Comparing equilibrium advertising spending by the high type under positivity (Lemma 16) to that under unrestricted word of mouth (Lemma 9) leads to the following proposition:

Proposition 8. *Advertising spending by the high type in a separating equilibrium that survives the Intuitive Criterion (Cho and Kreps 1987) refinement is lower in the game with compliments than in the game with unrestricted word of mouth.*

To understand the intuition, recall that when a low type mimics the high type, it gains more demand from influentials and this increases the number of followers that are matched with an influential who made a purchase. With unrestricted word of mouth, under mimicking followers who would have been otherwise exposed to silence instead hear either a positive or

negative experience, both of which lead to more favorable beliefs than under silence. In this model of compliments, since influentials do not share negative experiences, under mimicking more followers continue to encounter silence, which lowers the mimicking gains. Further, when $r_p < 1$, some of followers who would have otherwise heard a positive experience would now be exposed to silence, which yields less favorable beliefs. Taken together, the gains from mimicking for a low type are weaker, which imply that the high type requires a smaller advertising amount in equilibrium to credibly signal its type.

To see how advertising changes with the extent of positivity in word of mouth (r_p), it is useful to first study the effect of positivity on follower expected quality under silence:

Lemma 17. *Follower expected quality under silence ($E[q_0]_s = \frac{(1-H^2r_p)H+(1-L^2r_p)L}{1-H^2r_p+1-L^2r_p}$) in the presence of positivity is always decreasing in the level of positivity (r_p).*

As positivity increases, encountering silence is less likely when experiences are positive. Thus, more positivity implies that not hearing about an experience is more likely associated with the influential either not having bought or having a negative experience, both of which are more likely under a low type. Consequently, follower expected quality under silence decreases as positivity increases. The effects of positivity on the equilibrium advertising spending by the high type are described in the following proposition:

Proposition 9. *Advertising spending by the high type in a separating equilibrium satisfying the Intuitive Criterion (Cho and Kreps 1987) refinement in the game with compliments is always increasing in the extent of word of mouth positivity (r_p).*

As positivity increases, some followers who would have otherwise been exposed to silence are now exposed to compliments. Follower expected utility upon hearing a compliment is higher than that under silence. This increases the gains from mimicking, implying a greater advertising spending by the high type to credibly signal its quality.

7 Concluding Remarks And Future Research Directions

This research provides four main insights on how a firm might strategically adjust its advertising when faced with consumers who also learn about quality from their interactions with other consumers. First, when consumers learn, more money burns. This result is robust to consumers learning either via observations of initial purchases and/or repeat purchases, or them learning via word of mouth by sharing experiences or enjoyment/surplus. Specifically, in the presence of consumer interactions via either observational learning or word of mouth, it may be optimal for a firm to increase its advertising spending to credibly signal high quality. This is because consumer interactions can increase the potential benefits to a low quality firm from pretending to be of high quality. By mimicking a high quality firm, a low quality firm not only obtains incremental demand from early consumers, but these early consumers may positively influence the behavior of later consumers. This is somewhat counterintuitive; as one would expect a low quality firm to provide more negative experiences to early consumers, hence negatively impacting the behavior of later consumers. The above analysis shows that this may not always be the case. For instance, in the presence of word of mouth, even negative experiences can favorably influence later consumers. Their mere existence may convey to later consumers that early consumers considered the firm's quality to be sufficiently good for them to make a purchase. Hence, the stronger benefits from mimicking for a low quality firm, which arise when consumers share information via observation learning or word of mouth, may force a high quality firm to spend more on advertising. This is an important and novel insight from this work, particularly because conventional wisdom may argue that observational learning, word of mouth, and advertising might work as substitutes, since all three can provide information regarding quality to consumers. Consequently, a high quality firm should have to spend less on advertising as it would benefit from observational learning or positive word of mouth. However, this analysis shows that observational learning and word of mouth, rather than softening the need for spending on advertising, may require a high quality firm to allocate more resources to these quality signaling efforts.

The second main insight is derived from analyzing a commonly observed pattern of word of mouth behavior where not every consumer shares her experiences. Compared to the case where all consumers share their experiences, here a high quality firm may be better served by reducing its investment in advertising, and in certain conditions – quite significantly. This is because under-reporting softens the gain to a low quality firm from pretending to be of a high quality, due to two reasons. First, with under-reporting, when a low quality firm mimics a high quality one, fewer later consumers are exposed to word of mouth. Second, beliefs for these consumers when they do not hear about any experience are lower than if they actually hear about a positive or negative experience. Hence, to the extent that not all experiences are shared, the need for spending more in advertising in the presence of word of mouth is softened.

The third insight is obtained from the analysis of settings where consumers differentially share positive versus negative experiences. In contexts where there is positivity – i.e., positive experiences are more likely to be shared than negative ones – a firm’s optimal advertising spending increases with greater positivity. This is because with more positivity, mimicking the high type is more attractive to the low type firm when a greater proportion of later consumers hear these compliments.

Finally, the fourth main insight is derived from studying markets where there is negativity – i.e., negative experiences are more likely to be shared than positive ones. Here, when the high and low quality types are not too different from each other, as more negative experiences are shared, it is optimal for a high quality firm to spend more on advertising. This is because as the sharing of negative experiences increases, some late consumers who would have otherwise been exposed to no experiences are now exposed to complaints. When the high and low quality types are not too different from each other, hearing complaints instead of no experience leads to more favorable beliefs for late consumers. The implication of this result is that, when quality differences across types are not too strong, as consumers share more complaints, a high quality firm may need to allocate more resources to its advertising

quality signaling efforts.

The models analyzed in this paper come with their limitations, and thus there are several avenues for future research in this area. First and foremost, the current setup could be extended to consider how consumer interactions affect not only advertising but also pricing decisions aimed at signaling quality. Prices could be endogenously determined every period, or alternatively prices may be fixed to begin with but vary in subsequent periods. Further, in terms of belief formation, the analysis in this paper relies on a rational Bayesian updating process for follower beliefs as a function of firm communication and interactions with influentials. This learning process could potentially be more behaviorally nuanced. Incorporating behavioral updating mechanisms into the consumers' learning process might be an interesting direction for future research.

Another behavioral aspect of word of mouth is that sometimes experiences shared may not be based on actual consumption (e.g., Anderson and Simester 2014). Future research can consider the biases introduced due to the existence of such word of mouth in the mix of shared experiences. In addition, in this research information sharing via word of mouth is modeled as consumers interacting with either one or all peers. These assumptions could be further generalized in future research to investigate how the extent of information sharing among consumers affects learning and inference.

Furthermore, in this paper, advertising is the instrument used by the firm to signal its quality. In some situations, observational learning and word of mouth may be the only sources of information which are selectively accessible. Such situations would call for models where word of mouth could potentially become the signaling mechanism rather than advertising, and advertising becomes the mass medium instead. Future research can investigate such model formulations.¹⁸ Another important aspect that could be considered in future research is signal reach (i.e, not only the types of consumers that see the ads, but also the

¹⁸The authors thank an anonymous reviewer for his/her comments about a long-term equilibrium and word of mouth as a signaling mechanism.

number of consumers exposed to the ads).¹⁹ New models can explore the implications of reach and advertising costs – i.e., both the quality signaling as well as demand enhancing effects of advertising.

In terms of the game formulation - particularly dynamics and players - there are several possible extensions. For example, rather than a two-period model, a long-term game where consumers learn about the product's quality either via advertising, word of mouth or observational learning could be studied, leading to additional interesting outcomes. This analysis has considered a signaling game between a single firm and heterogeneous influentials and followers. It might be interesting to consider the implications of word of mouth and observational learning on advertising in the context of multiple firms, to investigate the role of inter-firm competition (e.g., Villas-Boas 2006). Finally, a related issue to consider would be an endogenous, rather than exogenous, assignment of consumers into influentials and followers, or early adopters versus late adopters. It may also be interesting to consider utility based incentives for individual consumers to engage in sharing behavior when modeling the transmission of word of mouth (e.g., Yoganarasimhan 2012; Iyer and Katona 2016).

Notwithstanding these additional extensions, this work indicates that learning via interactions among consumers should lead a high quality firm to systematically shift its advertising strategy in order to credibly signal its type.

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¹⁹In an extension, the firm's advertising spending is modeled to be a convex function of the reach of its signal among the influentials. In such a model, under certain conditions such as when the costs of advertising are high, no firm spends any money on advertising. Under other conditions, the result that advertising is higher in the presence of word of mouth than without, continues to hold. This analysis is available from the authors upon request.

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Appendix

This appendix provides mathematical proofs for all the lemmas in the paper. Proofs for all propositions are straightforward.

Proof for Lemma 1:

Assume that a pooling equilibrium exists. In any such equilibrium, let a firm of either type spend a common amount A^{pool} on advertising. Given fixed exogenous price p and zero marginal cost, in this equilibrium profit π_i^{pool} for firm type i can be written as $\pi_i^{pool} = p \cdot (\text{demand from influentials in period 1}) + p \cdot (\text{demand from influentials in period 2}) + p \cdot (\text{demand from followers in period 2}) - A^{pool}$. Consider $H < p \leq 1$ (the analysis for $1 < p < 1 + L$ yields different demand functions but otherwise follows analogous steps).

On the pooling equilibrium path, in period 1, influentials' expectations of the firm's type remain unchanged. An influential buys if $v + E[q_0] - p > 0$. Thus, the fraction of influentials who decide to buy in period 1 is $\int_{p-E[q_0]}^1 dv = 1 + E[q_0] - p$ leading to a demand of $1 + E_0[q_0] - p$ under either type, where $E_0[q_0] \equiv \kappa H + (1 - \kappa)L$.

Influentials with baseline utility $v > p - E_0[q_0]$ buy in period 1. In this group, those with $v > p$ repeat their purchase regardless of their consumption experience, since $\min_{q_0} v + q_0 - p = v - p > 0$. The remaining $p - p - E_0[q_0] = E_0[q_0]$ influentials repeat their purchase only if they have a positive consumption experience, which happens with probability q_0 . Accordingly, total demand from influentials in period 2 is $1 - p + E_0[q_0]q_0$. Followers enter the market in period 2, but receive no additional information. Hence they rely on their prior quality beliefs, and therefore a proportion $1 + E_0[q_0] - p$ of the followers make a purchase. Profits for the two types in a potential pooling equilibrium can be written as:

$$\pi_H^{pool} = p(1 + E_0[q_0] - p) + p(1 - p + E_0[q_0]H) + p(1 + E_0[q_0] - p)s - A^{pool}$$

$$\pi_L^{pool} = p(1 + E_0[q_0] - p) + p(1 - p + E_0[q_0]L) + p(1 + E_0[q_0] - p)s - A^{pool}$$

The rest of the proof shows that any such proposed pooling equilibrium does not survive the

Intuitive Criterion (Cho and Kreps 1987) refinement.

Denote by $A^{pooldev} \neq A^{pool}$ the advertising amount corresponding to a deviation from a pooling equilibrium, and $\pi_{q_0}^{pooldev}$ the corresponding profits obtained by type q_0 , where $q_0 \in \{L, H\}$. Let $\Theta^{**}(A^{pooldev})$ denote the set of all firm types for whom $A^{pooldev}$ is not equilibrium dominated, i.e. $\Theta^{**}(A^{pooldev}) \equiv \{q_0 \in \{L, H\} | \pi_{q_0}^{pool} \leq \pi_{q_0}^{pooldev}\}$. In other words, the value of $A^{pooldev}$ determines the types that belong to this set.

In this setting, there always exist some values of $A^{pooldev}$ such that $L \notin \Theta^{**}(A^{pooldev})$. This is because the most favorable off-the-equilibrium path belief that influentials could hold upon observing $A^{pooldev}$ is that the firm type is H . Followers do not observe advertising spending, hence they only rely on their prior quality beliefs.²⁰ Under this most favorable belief, deviation profit for the L type is $\pi_L^{pooldev} = p((1-p+H) + (1-p) + HL + (1-p + E_0[q_0])s) - A^{pooldev}$. This deviation profit is equilibrium dominated (i.e., $\pi_L^{pooldev} < \pi_L^{pool}$) if $A^{pooldev} > A^{pool} + p(H-L)(1+L)(1-\kappa)$. Hence, $L \notin \Theta^{**}(A^{pd})$ when $A^{pooldev} > A^{pool} + p(H-L)(1+L)(1-\kappa)$.

For $A^{pooldev} > A^{pool} + p(H-L)(1+L)(1-\kappa)$, a deviation by the H type is not equilibrium dominated if $\pi_H^{pooldev} > \pi_H^{pool}$. Per the Intuitive Criterion (Cho and Kreps 1987), when influentials observe $A^{pooldev} > A^{pool} + p(H-L)(1+L)(1-\kappa)$, they assign zero probability to the type being L . Therefore, $\pi_H^{pooldev} = p((1+H-p) + 1-p+H^2 + (1+E_0[q_0]-p)s) - A^{pooldev}$. Further, $\pi_H^{pooldev} > \pi_H^{pool}$ if $A^{pooldev} < A^{pool} + p(H-L)(1+H)(1-\kappa)$. Hence, for values of $A^{pooldev}$ such that $A^{pool} + p(H-L)(1+L)(1-\kappa) < A^{pooldev} < A^{pool} + p(H-L)(1+H)(1-\kappa)$, $\Theta^{**}(A^{pooldev}) = \{H\}$.

Given $0 < L < H < 1$ and $0 < \kappa < 1$ and $p > 0$, there always exist values of $A^{pooldev}$

²⁰Nevertheless, it is possible to determine follower beliefs for the advertising levels (even though they would not affect their choices). In the case of a pooling equilibrium, by definition, spending by both types is the same $A_H^* = A_L^* = A^{pool}$. Hence, follower beliefs in a pooling equilibrium are $p(A_H = A^{pool}) = p(A_L = A^{pool}) = 1$ and, of course, $p(A_H \neq A^{pool}) = p(A_L \neq A^{pool}) = 0$. Therefore, they rationally expect the advertising level to be equal to A^{pool} with probability 1.

In the case of the separating equilibria that will be discussed in the subsequent lemmas, each type spends a different amount denoted by A_H^* and A_L^* , where $A_H^* \neq A_L^*$. Hence, followers rationally expect only these two possible values for the advertising spending (which they don't observe) and, thus their beliefs about the advertising level are given by: $p(A = A_H^*) = p(q_0 = H) = \kappa$ and $p(A = A_L^*) = p(q_0 = L) = 1 - \kappa$.

satisfying the above conditions under which the H type is always willing to deviate from a pooling equilibrium, whereas the L type is not.

Thus, any pooling equilibrium that exists does not survive the Intuitive Criterion (Cho and Kreps 1987) refinement. ■

Proof for Lemma 2:

Consider $H < p \leq 1$ (the analysis for $1 < p < 1 + L$ yields different demand functions but otherwise follows analogous steps). In a separating equilibrium, neither type has any incentive to deviate and mimic the other. Mimicking is not profitable for the H type if $\pi_H \geq p((1 + L - p) + (1 - p) + LH + (1 + (\kappa H + (1 - \kappa)L) - p)s) - A_L$. Mimicking is not profitable for the L type if $\pi_L \geq p((1 + H - p) + (1 - p) + HL + (1 + (\kappa H + (1 - \kappa)L) - p)s) - A_H$. Together, both these conditions imply that neither type has any incentive to deviate and mimic the other if $p(H - L)(1 + L) \leq A_H - A_L \leq p(H - L)(1 + H)$.

The remainder of this proof has three main steps:

1. Given the above constraints on advertising, consider the existence of a separating equilibrium such that $A_L^* = 0$ and $A_H^* = p(H - L)(1 + L)$. On the equilibrium path, influentials' beliefs are $p(H|A = 0) = 0$ and $p(H|A = p(H - L)(1 + L)) = 1$. Off-the-equilibrium path, let influentials' beliefs be $p(H|0 < A < p(H - L)(1 + L)) = 0$ and $p(H|A > p(H - L)(1 + L)) = 1$. It is easy to see that this strategy and beliefs constitute a perfect Bayesian equilibrium of this signaling game.
2. To demonstrate that the equilibrium proposed in Step 1 above survives the Intuitive Criterion (Cho and Kreps 1987):
 - (a) Let the set of types that would have an incentive to deviate and spend $A \neq \{A_L^*, A_H^*\}$ on advertising under the most favorable off-the-equilibrium path beliefs be represented by $\Theta^{**}(A)$. It is easy to see that $\Theta^{**}(A) = \emptyset$ for $A > A_H^*$. For $A \in (0, A_H^*)$, if influentials' beliefs were such that $p(H|A) = 1$, then the high type has an incentive to deviate,

since it obtains the same demand but spends less on advertising; and the low type also has an incentive to deviate, since spending such an amount on advertising permits it to obtain the demand corresponding to the high type, leading to a higher profit as compared to the equilibrium outcome. Hence $\Theta^{**}(A) = \{L, H\}$, for $A \in (0, A_H^*)$.

- (b) Determine the subset of types in $\Theta^{**}(A)$ that have an incentive to deviate and spend $A \neq \{A_L^*, A_H^*\}$ on advertising under the least favorable off-the-equilibrium path beliefs. If influentials' beliefs were such that $p(L|A) = 1$, then deviation profits are weaker than equilibrium profits for either type. Thus, this subset is a null set.
- (c) Given (a) and (b) above, the proposed equilibrium survives the Intuitive Criterion.
- (d) Note that other separating equilibria may exist where the low type spends 0 and the high type spends $A^* = p(H - L)(1 + L)$, but influentials hold alternative off-the-equilibrium path beliefs.

3. Finally, analyze whether any other choice of A_L and A_H yields a perfect Bayesian equilibrium and survives the Intuitive Criterion:

- (a) Consider a candidate separating equilibrium where the low type spends $A'_L > 0$. If instead of spending A'_L , it spent 0 on advertising, its minimum profits would be obtained when influentials believe that such spending was made only by the low type. Those profits would still be higher than those obtained after spending $A'_L > 0$, since in the latter case the low type would spend more without improving influentials beliefs. Hence, in equilibrium, the low type always spends 0 on advertising.
- (b) Now consider candidate separating equilibria where the low type spends 0 on advertising, while the high type spends $A'_H \neq A_H^*$. If $A'_H < A_H^*$, then this would not yield a separating equilibrium since the low type would have an incentive to mimic the spending of the high type. If instead $A'_H > A_H^*$, to support this as an equilibrium, one would need beliefs such that $p(H|A < A'_H) = 0$ and $p(H|A \geq A'_H) = 1$. Although

this would yield a perfect Bayesian separating equilibrium, it would not survive the Intuitive Criterion:

- i. First, consider deviations $A^{dev} \in (A_H^*, A'_H)$. It is easy to see that the only type that would be willing to deviate in this range of advertising spending is the high type, i.e. $\Theta^{**}(A) = \{H\}$.
- ii. Constraining off-the-equilibrium path beliefs such that only the high type could have spent amounts $A^{dev} \in (A_H^*, A'_H)$, yields deviation profits that are always higher than those obtained when the high type spends A'_H .
- iii. Hence, any equilibrium with $A'_H > A_H^*$ does not survive the Intuitive Criterion. ■

Proof for Lemma 3: For a separating equilibrium to exist, neither firm type should have an incentive to deviate and mimic the other type. Consider $H < p \leq 1$ (the analysis for $1 < p < 1+L$ yields different demand functions but otherwise follows analogous steps). For a high type, deviation is not profitable if $\pi_H \geq p(1+L-p)+p(1-p+LH)+p(1+L-p)s-A_L$. For a low type, deviation is not profitable if $\pi_L \geq p(1+H-p)+p(1-p+HL)+p(1+H-p)s-A_H$. Together, these two conditions imply $p(H-L)(1+L+s) \leq A_H - A_L \leq p(H-L)(1+H+s)$. Applying the Intuitive Criterion and following arguments analogous to those in the proof of Lemma 2, it is easy to verify that all separating equilibria that survive this refinement are such that $A_L^* = 0$ and $A_H^* = p(H-L)(1+L+s)$. ■

Proof for Lemma 4: From Bayes' rule:

$$E[q_0]_p = \frac{(1+H-p)\kappa}{(1+H-p)\kappa+(1+L-p)(1-\kappa)}H + \frac{(1+L-p)(1-\kappa)}{(1+H-p)\kappa+(1+L-p)(1-\kappa)}L$$

$$E[q_0]_{np} = \frac{(p-H)\kappa}{(p-H)\kappa+(p-L)(1-\kappa)}H + \frac{(p-L)(1-\kappa)}{(p-H)\kappa+(p-L)(1-\kappa)}L$$

It is straightforward to verify $E[q_0]_p > \kappa H + (1-\kappa)L > E[q_0]_{np}$. ■

Proof for Lemma 5: For a separating equilibrium to exist, neither firm type should have

an incentive to deviate and mimic the other type. Consider $H < p \leq 1$ (the analysis for $1 < p < 1 + L$ yields different demand functions but otherwise follows analogous steps). For a high type, deviation is not profitable if $\pi_H \geq p(1 + L - p) + p(1 - p + LH) + p((1 + L - p)(1 + E[q_0]_p - p) + (p - L)(1 + E[q_0]_{np} - p))s - A_L$. For a low type, deviation is not profitable if $\pi_L \geq p(1 + H - p) + p(1 - p + HL) + p((1 + H - p)(1 + E[q_0]_p - p) + (p - H)(1 + E[q_0]_{np} - p))s - A_H$. Applying the Intuitive Criterion and following arguments analogous to those in the proof of Lemma 2, it is easy to verify that all separating equilibria that survive this refinement are such that $A_L^* = 0$ and $A_H^* = p(H - L)(1 + L + (E[q_0]_p - E[q_0]_{np})s)$. ■

Proof for Lemma 6: From Bayes' rule:

$$E[q_0]_r = \frac{(1-p+H^2)\kappa}{(1-p+H^2)\kappa+(1-p+L^2)(1-\kappa)}H + \frac{(1-p+L^2)(1-\kappa)}{(1-p+H^2)\kappa+(1-p+L^2)(1-\kappa)}L$$

$$E[q_0]_{nr} = \frac{(1-(1-p+H^2))\kappa}{(1-(1-p+H^2))\kappa+(1-(1-p+L^2))(1-\kappa)}H + \frac{(1-((1-p+L^2))(1-\kappa))}{(1-(1-p+H^2))\kappa+(1-(1-p+L^2))(1-\kappa)}L$$

It is straightforward to verify $E[q_0]_r > \kappa H + (1 - \kappa)L > E[q_0]_{nr}$. ■

Proof for Lemma 7: For a separating equilibrium to exist, neither firm type should have an incentive to deviate and mimic the other type. Consider $H < p \leq 1$ (the analysis for $1 < p < 1 + L$ yields different demand functions but otherwise follows analogous steps). For a high type, deviation is not profitable if $\pi_H \geq p(1 + L - p) + p(1 - p + LH) + p((1 - p + LH)(1 + E[q_0]_r - p) + (1 - (1 - p + LH))(1 + E[q_0]_{nr} - p))s - A_L$. For a low type, deviation is not profitable when $\pi_L \geq p(1 + H - p) + p(1 - p + HL) + p((1 - p + HL)(1 + E[q_0]_r - p) + (1 - (1 - p + HL))(1 + E[q_0]_{nr} - p))s - A_H$. Applying the Intuitive Criterion and following arguments analogous to those in the proof of Lemma 2, it is easy to verify that all separating equilibria that survive this refinement are such that $A_L^* = 0$ and $A_H^* = p(H - L)(1 + L + L(E[q_0]_r - E[q_0]_{nr})s)$. ■

Proof for Lemma 8: From Bayes' rule, $E[q_0]_+ = \frac{(1+H-p)H\kappa H+(1+L-p)L(1-\kappa)L}{(1+H-p)H\kappa+(1+L-p)L(1-\kappa)}$,

$E[q_0]_- = \frac{(1+H-p)(1-H)\kappa H+(1+L-p)(1-L)(1-\kappa)L}{(1+H-p)(1-H)\kappa+(1+L-p)(1-L)(1-\kappa)}$ and $E[q_0]_s = \frac{(p-H)\kappa H+(p-L)(1-\kappa)L}{(p-H)\kappa+(p-L)(1-\kappa)}$; for $H < p \leq 1$ it is straightforward to verify that $E[q_0]_+ > E[q_0]_- > E[q_0]_s$. ■

Proof for Lemma 9: For a separating equilibrium to exist, neither firm should have an incentive to deviate. For a high type, deviation is not profitable if $\pi_H \geq p(1+L-p)+p(1-p+LH)+p((1+L-p)(H(1+E[q_0]_+-p)+(1-H)(1+E[q_0]_- -p))+(p-L)(1+E[q_0]_s -p))s - A_L$. For a low type, deviation is not profitable if $\pi_L \geq p(1+H-p)+p(1-p+HL)+p((1+H-p)(L(1+E[q_0]_+-p)+(1-L)(1+E[q_0]_- -p))+(p-H)(1+E[q_0]_s -p))s - A_H$. Applying the Intuitive Criterion and following arguments analogous to those in the proof of Lemma 2, it is easy to verify that all separating equilibria that survive this refinement are such that $A_L^* = 0$ and $A_H^* = p(H-L)(1+L+s(L(E[q_0]_+ - E[q_0]_s) + (1-L)(E[q_0]_- - E[q_0]_s)))$. ■

Proof for Lemma 10: For a separating equilibrium to exist, neither firm type should have an incentive to deviate and mimic the other type. Consider $H < p \leq 1$ (the analysis for $1 < p < 1+L$ yields different demand functions but otherwise follows analogous steps). For a high type, deviation is not profitable if $\pi_H \geq p(1+L-p)+p((1-p)+LH)+p((1+L-p)(H(1+E[q_0]_{h+}-p)+(1-H)(1+E[q_0]_{h-}-p))+(p-L)(1+E[q_0]_{nop}-p))s - A_L$. For a low type, deviation is not profitable if $\pi_L \geq p(1+H-p)+p((1-p)+HL)+p((1+L-p)(L(1+E[q_0]_{h+}-p)+(1-L)(1+E[q_0]_{h-}-p))+(H-L)(1+H-p)+(p-H)(1+E[q_0]_{nop}-p))s - A_H$. Applying the Intuitive Criterion and following arguments analogous to those in the proof of Lemma 2, it is easy to verify that all separating equilibria that survive this refinement are such that $A_L^* = 0$ and $A_H^* = p(H-L)(1+L+(H-E[q_0]_{nop})s)$. ■

Proof for Lemma 11: $\frac{\partial}{\partial(1-r)}\left(\frac{(1-Hr)H+(1-Lr)L}{1-Hr+1-Lr}\right) = \frac{(H-L)^2}{(1-Hr+1-Lr)^2} > 0$. ■

Proof for Lemma 12: In this game, $E[q_0]_+ = \frac{H^3+L^3}{H^2+L^2}$, $E[q_0]_- = \frac{H^2(1-H)+L^2(1-L)}{H(1-H)+L(1-L)}$ and

$E[q_0]_s = \frac{(1-Hr)H+(1-Lr)L}{1-Hr+1-Lr}$. Profit for the high type can be written as $\pi_H = H + H^2 + H^2rE[q_0]_+ + H(1-H)rE[q_0]_- + (1-Hr)E[q_0]_s - A_H$, whereas profit for the low type can be written as $\pi_L = L + L^2 + L^2rE[q_0]_+ + L(1-L)rE[q_0]_- + (1-Lr)E[q_0]_s - A_L$. For a separating equilibrium to exist, neither firm type should have an incentive to deviate and mimic the other type. For a high type, deviation is not profitable if $\pi_H \geq L + LH + L HrE[q_0]_+ + L(1-H)rE[q_0]_- + (1-Lr)E[q_0]_s - A_L$. For a low type, deviation is not profitable if $\pi_L \geq H + HL + H LrE[q_0]_+ + H(1-L)rE[q_0]_- + (1-Hr)E[q_0]_s - A_H$. Applying the Intuitive Criterion and following arguments analogous to those in the proof of Lemma 2, it is easy to verify that all separating equilibria that survive this refinement are such that $A_L^* = 0$ and $A_H^* = (H-L)(1+L+LrE[q_0]_+ + (1-L)rE[q_0]_- - rE[q_0]_s)$, i.e. $A_H^* = (H-L)(1+L+r(L(E[q_0]_+ - E[q_0]_s) + (1-L)(E[q_0]_- - E[q_0]_s)))$. ■

Proof for Lemma 13: For $0 \leq r_n < 1$ it follows that $\frac{(1-H(1-H)r_n)H+(1-L(1-L)r_n)L}{1-H(1-H)r_n+1-L(1-L)r_n} > \frac{(1-H)H+(1-L)L}{1-H+1-L}$. ■

Proof for Lemma 14: In this game, $E[q_0]_- = \frac{H(1-H)H+L(1-L)L}{H(1-H)+L(1-L)}$ and $E[q_0]_s = \frac{(1-H(1-H)r_n)H+(1-L(1-L)r_n)L}{(1-H(1-H)r_n)+(1-L(1-L)r_n)}$. Profit for the high type can be written as $\pi_H = H + H^2 + H(1-H)r_nE[q_0]_- + (1-H(1-H)r_n)E[q_0]_s - A_H$, whereas profit for the low type can be written as $\pi_L = L + L^2 + L(1-L)r_nE[q_0]_- + (1-L(1-L)r_n)E[q_0]_s - A_L$. For a separating equilibrium to exist, neither firm type should have an incentive to deviate and mimic the other type. For a high type, deviation is not profitable if $\pi_H \geq L + LH + L(1-H)r_nE[q_0]_- + (1-L(1-H)r_n)E[q_0]_s - A_L$. For a low type, deviation is not profitable if $\pi_L \geq H + HL + H(1-L)r_nE[q_0]_- + (1-H(1-L)r_n)E[q_0]_s - A_H$. Applying the Intuitive Criterion and following arguments analogous to those in the proof of Lemma 2, it is easy to verify that all separating equilibria that survive this refinement are such that $A_L^* = 0$ and $A_H^* = (H-L)(1+L+r_n(1-L)(E[q_0]_- - E[q_0]_s))$. Also note that $E[q_0]_s > E[q_0]_-$ when $1-H < L$. ■

Proof for Lemma 15: $\frac{\partial}{\partial r_n} \left(\frac{(1-H(1-H)r_n)H+(1-L(1-L)r_n)L}{1-H(1-H)r_n+1-L(1-L)r_n} \right) = \frac{(H-L)^2(H+L-1)}{(1-H(1-H)r_n+1-L(1-L)r_n)^2} < 0 \Rightarrow L < H < 1 - L. \blacksquare$

Proof for Lemma 16: In this game, $E[q_0]_+ = \frac{H^3+L^3}{H^2+L^2}$ and $E[q_0]_s = \frac{(1-H^2r_p)H+(1-L^2r_p)L}{(1-H^2r_p)+(1-L^2r_p)}$. Profit for the high type can be written as $\pi_H = H + H^2 + H^2r_pE[q_0]_+ + (1 - H^2r_p)E[q_0]_s - A_H$, whereas profit for the low type can be written as $\pi_L = L + L^2 + L^2r_pE[q_0]_+ + (1 - L^2r_p)E[q_0]_s - A_L$. For a separating equilibrium to exist, neither firm type should have an incentive to deviate and mimic the other type. For a high type, deviation is not profitable if $\pi_H \geq L + LH + L Hr_p E[q_0]_+ + (1 - L Hr_p) E[q_0]_s - A_L$. For a low type, deviation is not profitable if $\pi_L \geq H + HL + H L r_p E[q_0]_+ + (1 - H L r_p) E[q_0]_s - A_H$. Applying the Intuitive Criterion and following arguments analogous to those in the proof of Lemma 2, it is easy to verify that all separating equilibria that survive this refinement are such that $A_L^* = 0$ and $A_H^* = (H - L)(1 + L + r_p L(E[q_0]_+ - E[q_0]_s))$. \blacksquare

Proof for Lemma 17: $\frac{\partial}{\partial r_p} \left(\frac{(1-H^2r_p)H+(1-L^2r_p)L}{1-H^2r_p+1-L^2r_p} \right) = -\frac{(H-L)^2(H+L)}{(1-H^2r_p+1-L^2r_p)^2} < 0. \blacksquare$