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# Digital marketplace: The role of probabilistic selling strategies in the travel industry

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ABSTRACT

With the rise of the digital economy and the depression in the travel industry due to the Covid-19 pandemic, the feasibility and profitability of innovative products on digital platforms, such as probabilistic selling, have garnered significant public attention. This paper utilizes a Salop circular city model featuring a single digital platform, two sellers, and a continuum of consumers to investigate the impact of probabilistic selling on the profits of the digital platform and sellers. We demonstrate that when the consumption utility is relatively low and product differentiation is at a moderate level, the market is in partial coverage. Offering appropriate probabilistic selling products in such cases expands the market, benefiting the digital platform, sellers and consumers simultaneously. Our findings contribute to the existing research on probabilistic selling techniques, digital business models, and the travel industry (e.g., hotel, car rental, airline).

## 1. Introduction

Numerous online platforms are reevaluating their business models in response to the burgeoning digital economy. Leveraging the Internet and big data technologies, digital platforms enable centralized online display, sales, and transactions of products and services in the travel industry. Users can seamlessly search, compare, and purchase goods directly through the app or website, eliminating the need for offline purchases or manual services. This innovative intermediary business model not only offers new revenue streams for associated businesses but also fuels digital innovation within the supply chain.

Simultaneously, the concept of probabilistic selling is thriving on digital platforms. This practice involves concealing certain vital details from customers during the sale of goods and then revealing them at a later time. For instance, Expedia and Priceline, two of the largest travel agents in the world, are actively engaged in the probabilistic selling of hotel rooms. Consumers only know the star rating of the hotel and the general area in which the hotel is located before paying, and they only receive specific information about the hotel after payment. A similar strategy can be observed in the car rental market. Both Expedia and Hertz, a renowned car rental service provider, categorize all models in their car rental business into groups like economy models, business models, luxury models, SUVs, and more. In the notes of each category, the service provider marks "xxx (classic model in the category) or similar." Users do not know the exact car they will get until they pay. As shown above, the application of probabilistic selling is an effective way for companies to boost their bottom line and a strategic approach for attracting new users and managing inventory.

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In addition, several major Chinese airlines, including China Eastern Airline, China Express, and Hainan Airline, have introduced a new type of probabilistic flight ticket called "fly at will" ("Sui Xin Fei" in Chinese Pinyin). In the rules of this category of tickets, the customers only choose the departure locations and are not informed about the exact destinations and time of the flight. Such tickets have raised great public attention for their much lower-than-regular prices compared to those with scheduled itineraries. Recently, the Shanghai Symphony Orchestra introduced a novel series of "Blind Box Concerts" featuring the music of Mozart, applying probabilistic selling strategies. Specifically, this series includes 12 concerts that will perform Mozart's compositions, but the exact program is not revealed to the audience until they enter the concert hall on the day of the performance. This uncertainty generates greater interest and engagement from the audience. The first concert was sold out quickly, indicating a positive market reception. In contrast, some top orchestras still have less than 50% booking rates. These two innovative applications show the promising potential of probabilistic selling in the travel industry.

The sudden outbreak of the COVID-19 pandemic in 2020 prompted governments worldwide to impose restrictions on international and domestic travel, severely affecting multiple travel industries, such as hotels, car rentals, aviation and art performances. Take the hotel industry as an example. According to data from the American Hotel & Lodging Association (AHLA), hotel revenue in the US plummeted nearly 50% in 2020 compared to 2019, with business activity dropping to levels not seen in 30 years. The impact was even more severe than that of 9/11. Many hotels were forced to close or lay off staff, with job losses reaching as high as 8.5 million, according to the Bureau of Labor Statistics. Occupancy rates dropped to record lows of 24.5% in April 2020. Luxury hotels were hit especially hard, with revenue dropping by over 80% in the first half of 2020. Although 2021 showed some improvement, revenue was still 40% below 2019 levels. The industry projects a full recovery to pre-pandemic business levels, likely not occurring until 2023 or 2024. This provides the travel industry in general with very strong incentives to innovate in their selling strategies to make up for lost revenue.

This paper investigates the impact of the probabilistic selling mechanisms employed by online travel product sales platforms (e.g., Ctrip, Priceline.com) on their profits and social welfare. We employ a Salop circular city model with one digital platform, two sellers, and a continuum of consumers. Sellers A and B strategically set their deterministic selling prices. The digital platform has the option to offer a probabilistic selling product by mixing the deterministic selling products A and B. Each consumer has a unit demand and maximizes their utility by purchasing their preferred product. Products sold by two sellers offer him the same baseline consumption utility but different disutilities of differentiation, which are positively correlated with the distance from the consumer's location to the seller's. We study both the full and partial market coverage cases.

Our analysis reveals that when the market is fully covered, consumers on the whole Salop circle are attracted by the deterministic selling products by sellers A and B, and thus, the platform never offers probabilistic selling products. However, in cases of partial market coverage (i.e., no direct competition between the two sellers), offering probabilistic selling products with a moderate mixing probability and product differentiation at a lower price expands market demand without exerting any influence on the deterministic selling products. Therefore, it is a Pareto improvement, increasing the consumer surplus and the profits of the platform and both sellers simultaneously. In addition, the optimal mixing probability chosen by the platform also maximizes total social welfare. We show that the timing of the probabilistic selling decision and the number of sellers in the market do not influence the aforementioned result by considering different timing scenarios and extending to an N-seller model. These findings offer valuable insights for digital platforms and firms in their strategic marketing decisions, enriching the discourse on probabilistic selling techniques and digital business models.

Our paper is closely related to recent research on digital platforms, probabilistic selling, and the Salop circular model. We fill a critical gap in the literature by concurrently enhancing welfare for both platforms and affiliated sellers, specifically focusing on the intersection of digital platforms and probabilistic selling, a topic that has been insufficiently explored.

In the realm of online pairing platforms, which fall under the category of digital platform markets, Dukes and Liu (Dukes and Liu, 2016) illustrate how platforms balance customer search costs and seller profits, while Armstrong (Armstrong, 2006) delves into platform behavior in two-sided markets. Hagiu and Wright (Hagiu and Wright, 2015) provide an overview of multi-sided platform concepts and Rochet and Tirole (Rochet and Tirole, 2003) analyze platform competition. Prado (Prado, 2018) demonstrates how platforms can increase stickiness and revenue by dominating multiple markets, and Chen and Zhu (Chen and Zhu, 2021) examine optimal platform pricing strategies. Yin et al. (Yin et al., 2020) study user pricing challenges on video platforms.

In past research on probabilistic selling, scholars have mostly focused on inventory, consumer behavior, and pricing, with limited connections to platforms and seller revenue. Fay and Xie (Fay and Xie, 2008), Jerath et al. (Jerath et al., 2010), and Zhang and Cooper (Zhang and Cooper, 2008) analyze pricing under probabilistic selling. Wu and Jin (Wu and Jin, 2022) find that it can increase expected profits, and Mao et al. (Mao et al., 2020) explore retailer adoption considerations. Fay and Xie (Fay and Xie, 2010) compare advanced and probabilistic selling, and Zhang et al. (Zhang et al., 2015) examine quality variations. Huang and Yu (Huang and Yu, 2014) provide a customer perspective.

Specifically in the travel industry, Wang, Gal-Or, and Chatterjee (Wang et al., 2009) analyzed the application of the name-your-own-price channel, developing a model to investigate optimal pricing strategies for travel sellers using probabilistic selling. Their findings revealed that sellers' optimal pricing decisions are influenced by customer valuation heterogeneity, strategic customer fraction, and capacity. Furthermore, they showed that the name-your-own-price channel benefits the travel seller through price discrimination. Anderson and Xie (Anderson and Xie, 2012) studied opaque pricing in hotels, proposing a choice-based dynamic programming approach for setting opaque prices. Their research provides insights into probabilistic pricing strategies for the hospitality industry.

This paper is notable for applying the Salop circular city model to a market that may involve the probabilistic selling of products. In similar studies in the past, scholars preferred to use the Hotelling model for their research. Compared to the Hotelling model, the

Salop circular city model allows us to extend our study from two sellers to N sellers, which supports the robustness of our results. Li and Feng (Li and Feng, 2017) utilized the Salop circular city model to analyze commodity value sensitivity and the utility of the model itself. Empirical data used in their study affirmed the efficacy of the Salop circular city model, providing support for the present study. Basu (Basu et al., 2023) investigated enhancing channel efficiency by curbing retailers' bilateral marginal losses through heightened distribution intensity under the Salop circular city model. The study revealed that, under exclusive distribution, retailers adopt spatial monopoly pricing; in non-exclusive distribution, retailers face local competition. Li and Shuai (Li and Shuai, 2019) leveraged the Salop circular city model to explore price discrimination in monopolistic competition. They determined that, with a fixed number of enterprises, price discrimination can augment enterprise profits but potentially diminish consumer welfare compared to uniform pricing. Liu and Serfes (Liu and Serfes, 2007) examined both first-degree and third-degree price discrimination in circular city models with uniform and non-uniform distributions of consumers. Their analysis provided insights into the profitability of price discrimination strategies and their welfare implications. The paper also compared price discrimination outcomes between circular and linear city models.

The aforementioned work establishes a robust theoretical foundation for the research conducted in this paper. Nonetheless, our study differentiates itself from these studies in several key aspects. Firstly, a significant portion of the literature we reviewed primarily focuses on examining retailers or businesses utilizing digital platforms, while our research emphasizes the digital platform's strategic behavior, i.e., whether to offer probabilistic selling products. Secondly, the previous studies typically restrict attention to monopolistic organizations, whereas we extend our analysis to encompass oligarchic markets with multiple sellers. Furthermore, the existing body of research predominantly employed probabilistic selling to probe topics such as item pricing, the influence of probabilistic selling on customer decisions, and the interplay between consumer behaviors and the frequency of probabilistic selling employed by platforms or businesses. In contrast, we study how platforms can strategically leverage probabilistic selling to enhance their profitability.

The remainder of the paper is organized as follows. Section 2 establishes the foundation by conducting a literature review covering topics such as the digital platform market, probabilistic selling, and the Salop circular city model. Section 3 outlines our model setups. Sections 4 and 5 present and explain the equilibrium solutions when the market is fully and partially covered, respectively. In Section 6, we extend our baseline 2-seller model to the N-seller case. Finally, we conclude in Section 7.

## 2. Model setups

### 2.1. Salop circle model

Two sellers, A and B, sell products on a digital service platform (denoted by $P$) to consumers that are uniformly distributed along a Salop circle of circumference $L$. Each consumer has a unit demand. Sellers A and B are located on the Salop circle at points 0 and $\frac{L}{2}$, respectively, and offer identical products with the same baseline consumption utility $U$. The disutility that a product brings to a consumer due to the distance $d$ from the consumer's location is represented by the quadratic function $td^2$, where $t\,(>0)$ is the disutility per unit distance. The above assumptions can be understood as follows: the products sold by the two companies have horizontal differentiation (e.g., departure time, number of layovers, etc.), and the degree of differentiation is represented by $t$, where a larger $t$ indicates that consumers perceive greater differentiation between the products. To simplify the analysis, the production and selling costs of tickets are assumed to be zero.[1]

### 2.2. Deterministic selling of the products

The utility of a consumer located at point $x$ who purchases from sellers A and B, respectively, can be denoted by $u_A$ and $u_B$ and expressed as follows:

$$u_A = U - tx^2 - p_A \tag{1}$$

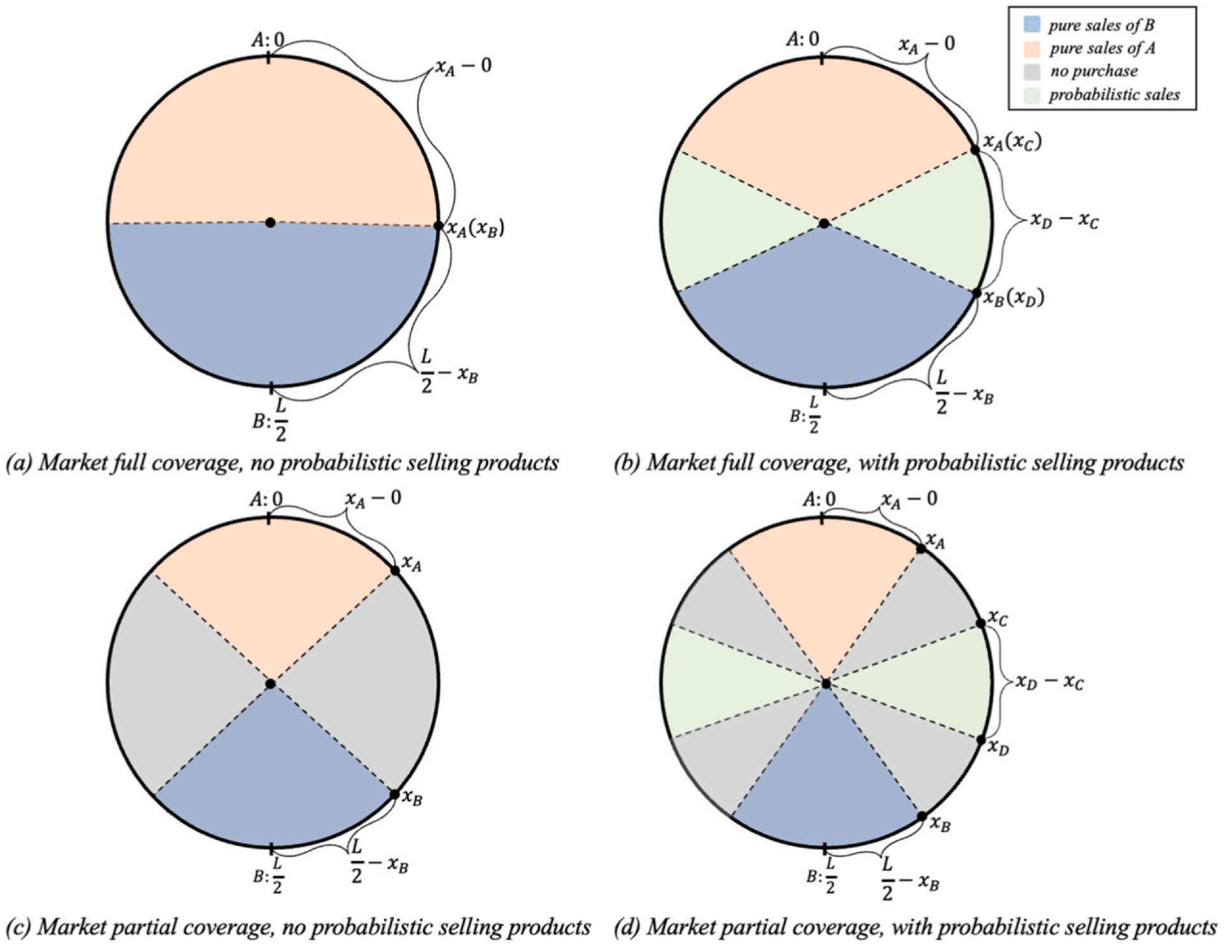
$$u_B = U - t\left(\frac{L}{2} - x\right)^2 - p_B \tag{2}$$

where $p_A$ and $p_B$ are the prices of the products sold by sellers A and B, respectively. The two equations above show that after paying the price $p_A$ ($p_B$), the consumer will obtain the specific product A (B), which we refer to as deterministic selling of the products.

### 2.3. Probabilistic selling products

The platform has an option to coordinate with sellers and offer probabilistic products. In this case, consumers pay a price $p_M$ that entitles them to a unit of the good produced by seller A with probability $a \in (0,1)$ and a unit of the good produced by seller B with probability $1 - a$, thus introducing uncertainty about the product the consumer will receive. The utility of a consumer purchasing a probabilistic selling product at point $x$ can be denoted by $u_M$ and expressed as follows:

[1] This assumption does not affect the results.




*(a) Market full coverage, no probabilistic selling products* *(b) Market full coverage, with probabilistic selling products*

*(c) Market partial coverage, no probabilistic selling products* *(d) Market partial coverage, with probabilistic selling products*

**Fig. 1.** the Salop circles with and without probabilistic selling products in a market with full and partial coverage.

$$u_M = a(U - tx^2) + (1 - a)\left[U - t\left(\frac{L}{2} - x\right)^2\right] - p_M \tag{3}$$

The term $a(U - tx^2)$ represents the utility obtained by the consumer when purchasing the product sold by seller A with a probability of $a$ and receiving a utility of $U - tx^2$. The term $(1 - a)[U - t(\frac{L}{2} - x)^2]$ represents the utility obtained by the consumer when purchasing the product sold by seller B with a probability of $1 - a$ and receiving a utility of $U - t(\frac{L}{2} - x)^2$. Eq. (3) can be further simplified as follows:

$$u_M = U - a(tx^2) - (1 - a)t\left(\frac{L}{2} - x\right)^2 - p_M \tag{4}$$

### 2.4. Full coverage and partial coverage

As is shown in Fig. 1, sellers A and B are located at points 0 and $\frac{L}{2}$. Without probabilistic selling products, consumers lying on the upper arc between points $x_A$ and $L - x_A$ and the lower arc between points $x_B$ and $L - x_B$ purchase products A and B, respectively. The market is fully covered when $x_A = x_B$, and partially covered when $x_A < x_B$. With probabilistic selling products, besides consumers purchasing deterministic products A and B, consumers that lie on the minor arcs between points $x_C$ and $x_D$, and points $L - x_D$ and $L - x_C$ purchase probabilistic selling products. No consumer purchases the probabilistic selling product when $x_C = x_D$.

### 2.5. Profits of sellers A, B and the platform

The revenue of deterministic selling products A, B and probabilistic selling products, $R_A$, $R_B$, and $R_M$, can be expressed as:

$$R_A = p_A{*}2{*}(x_A - 0) \tag{5}$$

$$R_B = p_B{*}2{*}\left(\frac{L}{2} - x_B\right) \tag{6}$$

$$R_M = p_M{*}2{*}(x_D - x_C) \tag{7}$$

For simplicity, we assume the platform gains $\rho$ proportion of all revenue as commission fees. Seller A gains $a$ proportion of the remaining probabilistic selling revenue (and hence, seller B gains $1 - a$ proportion of the remaining probabilistic selling revenue). The profits of sellers A, B and the platform, $\pi_A$, $\pi_B$ and $\pi_P$ can be expressed as:

$$\pi_A = (1 - \rho)(R_A + a{*}R_M) \tag{8}$$

$$\pi_B = (1 - \rho)[R_B + (1 - a){*}R_M] \tag{9}$$

$$\pi_P = \rho(R_A + R_B + R_M) \tag{10}$$

### 2.6. Timeline

We examine various timing scenarios presented in the following three cases. To comprehensively investigate the potential for introducing probabilistic selling products, we assume that the platform makes the decision of whether to introduce probabilistic selling products at different points in time.

**Case 1.** First, the platform decides whether to offer probabilistic selling products and determines the selling price $p_M$. Then, observing the platform's strategy, the sellers A and B simultaneously decide their deterministic selling prices, $p_A$ and $p_B$. Finally, the consumers make their purchasing choices.

**Case 2.** First, the sellers A and B simultaneously decide their deterministic selling prices, $p_A$ and $p_B$. Then, observing the sellers' strategies, the platform decides whether to offer probabilistic selling products and determines the selling price $p_M$. Finally, the consumers make their purchasing choices.

**Case 3.** First, the platform and sellers A and B simultaneously decide whether to offer probabilistic selling products and the selling price $p_M$, the selling price of deterministic selling products A and B $p_A$ and $p_B$ to maximize their own profits respectively. Then, the consumers make their purchasing choices.

## 3. Offering probabilistic selling products in a full coverage market

In this section, we present the case of full market coverage and show that when the degree of product differentiation is not too large, probabilistic selling does not occur in equilibrium, regardless of the timing of the platform's decision.

### 3.1. Equilibrium without probabilistic selling products

When there are only sellers A and B offering deterministic selling of products and the market is fully covered, from Eqs. (1) to (2), it can be observed that for the indifferent consumer located between sellers A and B, where $u_A = u_B$, their position $\bar{x} = x_A = x_B$ can be expressed as:

$$\bar{x} = \frac{-4p_A + 4p_B + L^2 t}{4Lt} \tag{11}$$

The following proposition presents the equilibrium solution and shows how the platform's expected profit is influenced by the level of product differentiation ($t$).

**Proposition 1.** *When* $0 < t \le \frac{16U}{5L^2}$, the market is fully covered. In the equilibrium without probabilistic selling products:

(1) *The sellers sell* products *A and B at prices* $p_A^* = p_B^* = \frac{L^2 t}{4}$.
(2) *Eonsumers lying on the upper and lower arcs between points* $\frac{L}{4}$ *and* $\frac{3L}{4}$ *purchase products A and B respectively.*
(3) *The platform's profit* $\pi_P = \frac{1}{4}L^3 t\rho$ *is increasing in the level of product differentiation* ($t$).

See the appendix for detailed proof.

From Proposition 1, the platform's expected profit increases with the level of product differentiation ($t$) for the products sold by sellers A and B. When $t$ increases, consumers perceive an increase in the differentiation between the products sold by sellers A and B, leading to weaker competition and an increase in both $p_A^*$ and $p_B^*$. Consequently, the total equilibrium revenue for products A and B increases, which in turn increases the commission fee earned by the platform.

### 3.2. Equilibrium with probabilistic selling products

In this subsection, we allow the platform to offer probabilistic selling products. We investigate whether the platform opts to provide probabilistic selling products, and whether a specific group of consumers selects these probabilistic selling products in Cases 1–3.

When consumers have the option to choose between deterministic and probabilistic selling products in the market, for the indifferent consumers satisfying $u_A = u_M$ and $u_B = u_M$, we can represent their positions $x_A$ and $x_B$ as follows:

$$x_A = \frac{4p_A - 4p_M + (-1+a)L^2 t}{4(-1+a)Lt} \tag{12}$$

$$x_B = \frac{4p_B - 4p_M + aL^2 t}{4aLt} \tag{13}$$

The following proposition provides the equilibrium prices and purchasing decisions for deterministic selling and probabilistic selling products.

**Proposition 2.** *When the market is fully covered,*

(1) The equilibrium prices of deterministic selling products A, B and probabilistic selling products $p_A^* = p_B^* = p_M^* = \frac{L^2 t}{4}$.
(2) Consumers lying on the upper and lower arcs between points $\frac{L}{4}$ and $\frac{3L}{4}$ purchase products A and B respectively.[2]

See the appendix for detailed proof.

From $x_A^* = x_B^* = x_C^* = x_D^* = \frac{L}{4}$, we find no consumers purchase the probabilistic selling products. From $p_A^* = p_B^* = p_M^* = \frac{L^2 t}{4}$, we find no incentives to lower the deterministic selling prices, comparing to the scenario without probabilistic selling products.

Proposition 2 implies the following corollary.

**Corollary 1.** *When the market is fully covered, the platform will not offer probabilistic selling products, and the equilibrium result is the same as in* Proposition 1.

It is worth noting that when $\frac{16U}{5L^2} < t \le \frac{16U}{3L^2}$, the market is still fully covered, but there is no market competition between sellers A and B. In this scenario, the equilibrium prices are $p_A^* = p_B^* = p_M^* = U - \frac{L^2 t}{16}$, and $x_A^* = x_B^* = x_C^* = x_D^* = \frac{L}{4}$, which aligns with the conclusion in Corollary 1: probabilistic selling products will have no market demand and will not generate higher profits for the platform.

## 4. Offering probabilistic selling products in a partial coverage market

In this section, we show partial market coverage when the degree of product differentiation is large enough. Our results illustrate that, in this case, offering probabilistic selling products at reduced prices could yield higher profits for the platform.

### 4.1. Equilibrium without probabilistic selling products

When there are only sellers A and B offering deterministic selling products and the market is partially covered, the indifferent consumers who do not buy products from sellers A or B satisfy $u_A = 0$ ($u_B = 0$), and their positions can be represented as follows:

$$x_A = \frac{\sqrt{-p_A + U}}{\sqrt{t}} \tag{14}$$

$$x_B = \frac{L}{2} - \frac{\sqrt{-p_B + U}}{\sqrt{t}} \tag{15}$$

The following proposition presents the equilibrium solution and shows how the platform's expected profit is influenced by the level of product differentiation ($t$) and the baseline consumption utility ($U$).

**Proposition 3.** *When* $t > \frac{16U}{3L^2}$, the market is partially covered. In the equilibrium without probabilistic selling products:

(1) *The sellers sell products A and B at prices* $p_A^* = p_B^* = \frac{2U}{3}$.
(2) *Consumers lying on the minor arc between points* $-\frac{\sqrt{U}}{\sqrt{3}\sqrt{t}}$ *and* $\frac{\sqrt{U}}{\sqrt{3}\sqrt{t}}$ *purchase products A and consumers lying on the minor arc between points* $\frac{L}{2} - \frac{\sqrt{U}}{\sqrt{3}\sqrt{t}}$ *and* $\frac{L}{2} + \frac{\sqrt{U}}{\sqrt{3}\sqrt{t}}$ *purchase products B.*
(3) *The platform's profit* $\pi_P = \frac{8U\sqrt{U}\rho}{3\sqrt{3}\sqrt{t}}$ *is decreasing in the level of product differentiation* ($t$) *and increasing in the baseline utility* ($U$).

[2] Proposition 4 is robust to different timing of the probabilistic selling decision. Thus, we prove the results in Cases 1–3.

See the appendix for detailed proof.

Proposition 3 highlights an interesting contrast to Proposition 1 under the condition of no probabilistic selling products. Specifically, it reveals that the platform's profit decreases as the level of product differentiation ($t$) increases. The reason is that with an increase in $t$, $x_A^*$ decreases while $x_B^*$ increases, leading to a contraction in the market demand for sellers A and B. As a result, the total equilibrium revenue for sellers A and B decreases, leading to a reduction in the platform's profit.

Furthermore, Proposition 3 also indicates that the platform's profit increases with an increase in the baseline utility ($U$), as an increase in $U$ leads to an increase in the market demand for sellers A and B. Consequently, the total equilibrium revenue for sellers A and B increases, leading to an increase in the platform's profit.

### 4.2. Equilibrium with probabilistic selling products

In this subsection, we allow the platform to offer probabilistic selling products but take the mixing probability $a$ as exogenously given. We investigate whether the platform opts to provide probabilistic selling products and whether a specific group of consumers selects these probabilistic selling products.

When the market is partially covered, the utilities of the marginal consumers who choose to buy deterministic products A and B satisfy $u_A = 0$ and $u_B = 0$. Thus, their positions are the same as given in the previous subsection by Eqs. (14) and (15). Additionally, the utilities of the marginal consumers who choose to buy probabilistic selling products satisfy $u_C = 0$ and $u_D = 0$. We can represent their positions as follows:

$$x_C = \frac{1}{2}\left(L - aL - \frac{\sqrt{-4p_M - aL^2t + a^2L^2t + 4U}}{\sqrt{t}}\right) \tag{16}$$

$$x_D = \frac{1}{2}\left(L - aL + \frac{\sqrt{-4p_M - aL^2t + a^2L^2t + 4U}}{\sqrt{t}}\right) \tag{17}$$

The following proposition provides the equilibrium prices and purchasing decisions for deterministic and probabilistic selling products.

**Proposition 4.** *When the market is partially covered:*

(1) *The equilibrium prices of deterministic selling products A, B are* $p_A^* = p_B^* = \frac{2U}{3}$ *and the price of probabilistic selling products is* $p_M^* = \frac{1}{6}(-aL^2t + a^2L^2t + 4U)$.

(2) *Consumers lying on the minor arc between points* $-\frac{\sqrt{U}}{\sqrt{3}\sqrt{t}}$ *and* $\frac{\sqrt{U}}{\sqrt{3}\sqrt{t}}$ *purchase products A, consumers lying on the minor arc between points* $\frac{L}{2} - \frac{\sqrt{U}}{\sqrt{3}\sqrt{t}}$ *and* $\frac{L}{2} + \frac{\sqrt{U}}{\sqrt{3}\sqrt{t}}$ *purchase products B, and consumers lying on the minor arc between points* $\frac{1}{2}(1-a)L - \frac{\sqrt{12U - 3(1-a)aL^2t}}{6\sqrt{t}}$ *and* $\frac{1}{2}(1-a)L + \frac{\sqrt{12U - 3(1-a)aL^2t}}{6\sqrt{t}}$ *and the minor arc between points* $\frac{1}{2}(1+a)L - \frac{\sqrt{12U - 3(1-a)aL^2t}}{6\sqrt{t}}$ *and* $\frac{1}{2}(1+a)L - \frac{\sqrt{12U - 3(1-a)aL^2t}}{6\sqrt{t}}$ *purchase probabilistic selling products.*[3]

See the appendix for detailed proof.

Proposition 4 provides two significant insights. First, when the market is not fully covered, there is no competition between the deterministic selling products offered by sellers A and B and the probabilistic selling products offered by the platform. Therefore, the prices of deterministic selling products remain unaffected by the platform's probabilistic selling strategy.

Second, the equilibrium price of probabilistic selling products offered by the platform is lower than the equilibrium prices of deterministic selling products offered by sellers A and B (i.e., $p_A^* = p_B^* > p_M^*$ for any $a \in (0,1)$ and $t > \frac{16U}{3L^2}$). This is due to the fact that in the partial coverage scenario, consumers far away from both sellers A and B would choose not to purchase the deterministic products A and B. The platform attracts certain consumers to opt for probabilistic selling products by strategically reducing its selling price.

In summary, the introduction of probabilistic selling products has the potential to expand market demand without exerting any influence on the deterministic selling products. Let $0 < x_A < x_C < x_D < x_B$ to guarantee that the probabilistic selling products will not compete with the deterministic selling products. We directly derive the following corollary.

**Corollary 2.** *When the market is partially covered, offering proper probabilistic selling products that satisfy the following condition (18) is a Pareto improvement.*

$$\begin{cases} \frac{1}{4} < a \le \frac{1}{2} \ \&\ and\ \&\ \frac{48U}{L^2 + 4aL^2 + 4a^2L^2} < t < -\frac{4U}{-aL^2 + a^2L^2} \\ \frac{1}{2} < a < \frac{3}{4} \ \&\ and\ \&\ \frac{48U}{9L^2 - 12aL^2 + 4a^2L^2} < t < -\frac{4U}{-aL^2 + a^2L^2} \end{cases} \tag{18}$$

*Consumer surplus (CS), profits of sellers A, B ($\pi_A$, $\pi_B$), the platform ($\pi_P$) and total social welfare (SW) increase by*

[3] Proposition 4 is robust to different timing of the probabilistic selling decision.

$$\Delta CS = CS_2 - CS_1 = \frac{[(-1+a)aL^2t + 4U]^{\frac{3}{2}}}{9\sqrt{3t}} > 0 \tag{19}$$

$$\Delta\pi_A = \pi_{A2} - \pi_{A1} = -\frac{a[(-1+a)aL^2t + 4U]^{\frac{3}{2}}(\rho - 1)}{3\sqrt{3t}} > 0 \tag{20}$$

$$\Delta\pi_B = \pi_{B2} - \pi_{B1} = \frac{(a-1)[(-1+a)aL^2t + 4U]^{\frac{3}{2}}(\rho - 1)}{3\sqrt{3t}} > 0 \tag{21}$$

$$\Delta\pi_P = \pi_{P2} - \pi_{P1} = \frac{[(-1+a)aL^2t + 4U]^{\frac{3}{2}}\rho}{3\sqrt{3t}} > 0 \tag{22}$$

$$\Delta SW = \frac{4[(-1+a)aL^2 + 4U]^{\frac{3}{2}}}{9\sqrt{3t}} > 0 \tag{23}$$

See the appendix for detailed proof.

From Corollary 2, we can deduce that under the condition in Eq. (18), indicating that the mixing probability ($a$) and the level of product differentiation ($t$) are all at moderate levels, offering probabilistic selling products enables the platform to increase the consumer surplus, profits of itself and two sellers. Specifically, when the mixing probability ($a$) is very high or very low, the probabilistic selling product closely resembles one of the deterministic selling products. Consequently, the platform cannot simultaneously attract both consumers who previously chose not to purchase deterministic selling products and consumers who previously chose to purchase deterministic selling products.

Furthermore, when the level of product differentiation ($t$) is either very high or very low, the implementation of probabilistic selling products will not increase the platform's profit. In the case of high product differentiation ($t$), if purchased probabilistic selling products do not align with consumer preferences, substantial disutility may result, dissuading consumers from buying such products and negating potential welfare improvement. Conversely, when product differentiation ($t$) is very low, the market will be in full coverage, as proved in Proposition 2, and probabilistic selling products will have no market value, meaning no consumers will purchase probabilistic selling products.

However, when the mixing probability ($a$) and the level of product differentiation ($t$) are both moderate (i.e. when the conditions (18) are met), the platform can attract more probabilistic selling product buyers without influencing the deterministic selling product buyers, thereby expanding the market demand, and increasing the total social welfare.

### 4.3. Endogenous mixing probability

In this subsection, we discuss how the platform endogenously determines the optimal mixing probability $a^*$, i.e., the probability that consumers will receive seller A's product when purchasing probabilistic selling products and examine the equilibrium results under the platform's optimal mixing probability.

**Proposition 5.** *The platform's optimal mixing probability is* $a^* = -\frac{1}{2} + \frac{2\sqrt{3U}}{\sqrt{L^2t}}$ or $\frac{3}{2} - \frac{2\sqrt{3U}}{\sqrt{L^2t}}$. At this optimal mixing probability:

(1) *The equilibrium prices for sellers A and B offering deterministic selling products are* $p_A^* = p_B^* = \frac{2U}{3}$, *and the platform's equilibrium price for probabilistic selling products is* $p_M^* = \frac{8U}{3} + L^2t(\frac{1}{8} - \frac{2\sqrt{U}}{\sqrt{3L^2t}})$.
(2) *When* $\frac{12U}{L^2} < t < \frac{64U}{3L^2}$, *offering probabilistic selling products increases the profits for both the platform and sellers A and B.*
(3) *The platform's profit decreases with an increase in the level of product differentiation ($t$) and increases with an increase in the baseline utility ($U$).*

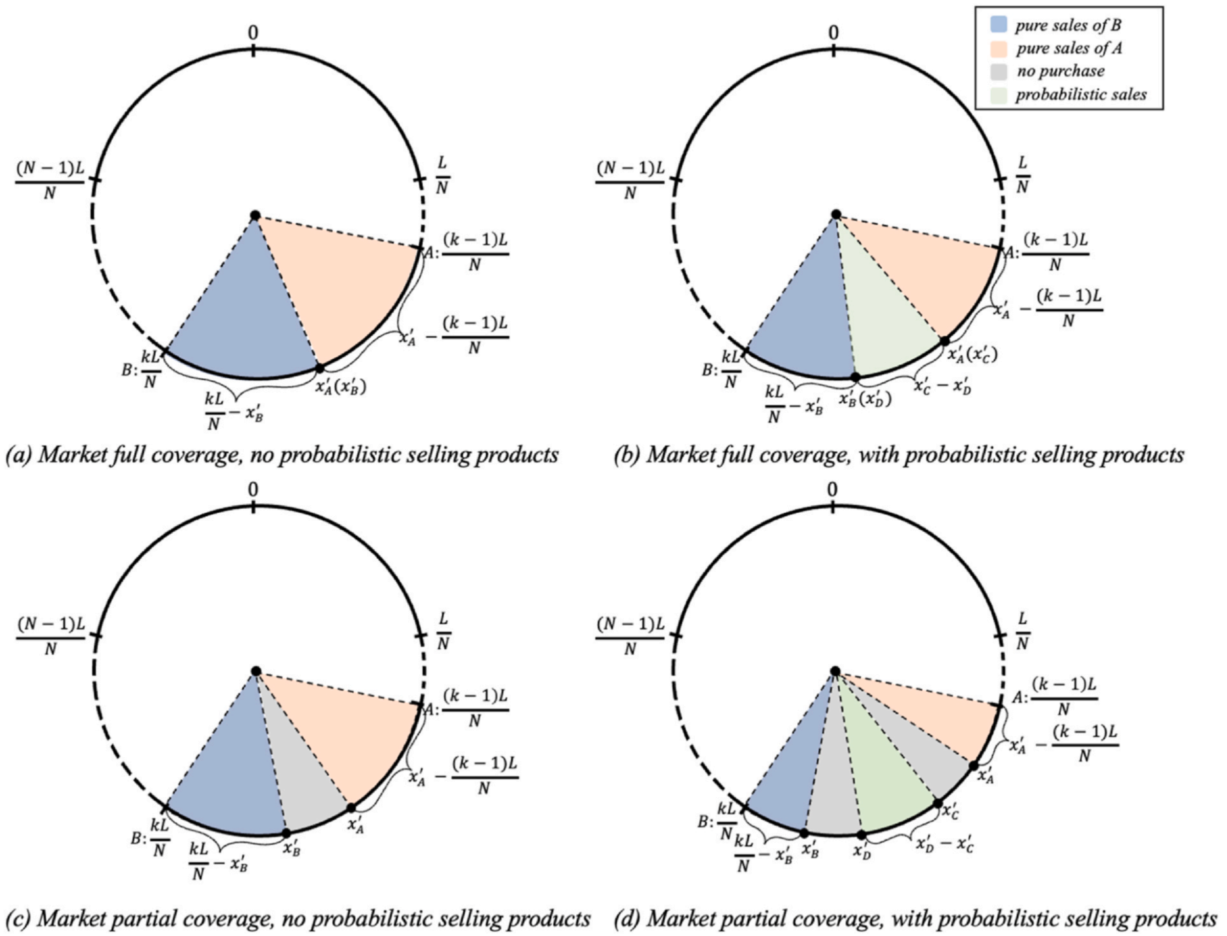
See the appendix for detailed proof.

At the optimal mixing probability $a^*$, we observe that the equilibrium price for probabilistic selling products decreases with an increase in the level of product differentiation ($t$) and increases with an increase in the baseline utility ($U$). Specifically, when the level of product differentiation ($t$) increases, the disutility incurred by consumers when probabilistic selling products do not match their preferences becomes more significant, leading to a decrease in the demand for probabilistic selling products. To attract more consumers to purchase probabilistic selling products, the platform lowers the equilibrium price. On the other hand, when the baseline utility ($U$) increases, the disutility from mismatches between consumer preferences and probabilistic selling products is compensated by the consumption utility. Consequently, the equilibrium price for probabilistic selling products rises.

In the following corollary, we further show that the above mixing probability $a^*$ that maximizes the revenue of probabilistic selling products (thus the profits of the platform and two sellers) is also optimal for consumers and the whole society.

**Corollary 3.** *The mixing probability* $a^*$ concurrently maximizes consumer surplus and the total social welfare.

See the appendix for detailed proof.




(a) Market full coverage, no probabilistic selling products (b) Market full coverage, with probabilistic selling products

(c) Market partial coverage, no probabilistic selling products (d) Market partial coverage, with probabilistic selling products

**Fig. 2.** the Salop circles with N sellers.

## 5. Salop model with N sellers

We extend the baseline 2-seller model in Section 2 to a N-seller model. To simplify the problem, we assume that $N$ sellers are evenly distributed on the Salop circle. Without loss of generality, our attention shall be directed solely towards any arbitrarily selected adjacent pair of sellers, along with the customers distributed between these two sellers.

As is shown in Fig. 2, suppose two adjacent sellers A and B are located at points $\frac{(k-1)L}{N}$ and $\frac{kL}{N}$. Without probabilistic selling products, consumers lying on the minor arc between points $\frac{(k-1)L}{N}$ and $x'_A$ and the minor arc between points $x'_B$ and $\frac{kL}{N}$ purchase products A and B, respectively. The market is fully covered when $x'_A = x'_B$, and partially covered when $x'_A < x'_B$. With probabilistic selling products, besides consumers purchasing products A and B, consumers lying on the minor arc between points $x'_C$ and $x'_D$ purchase probabilistic selling products that mix products A and B. No consumer purchases the probabilistic selling product when $x'_C = x'_D$.

For deterministic selling of products, the utility of a consumer located at position $x \in (\frac{(k-1)L}{N}, \frac{kL}{N})$on the circumference purchasing products A and B is:

$$u'_A = U - t\left[x - \frac{(k-1)L}{N}\right]^2 - p'_A \tag{24}$$

$$u'_B = U - t\left(\frac{kL}{N} - x\right)^2 - p'_B \tag{25}$$

For probabilistic selling products, suppose the consumer located at position $x$ has a probability $a \in (0,1)$ to obtain product A. Thus, his utility of the probabilistic selling product is:

$$u'_M = a\left[U - t\left(x - \frac{(k-1)L}{N}\right)^2\right] + (1-a)\left[U - t\left(\frac{kL}{N} - x\right)^2\right] - p'_M \tag{26}$$

Eq. (26) can be further simplified as follows:

$$u_M' = U - a\left[t\left(x - \frac{(k-1)L}{N}\right)^2\right] + (1-a)\left[t\left(\frac{kL}{N} - x\right)^2\right] - p_M' \tag{27}$$

The revenue of deterministic selling products A, B and probabilistic selling products purchased by consumers lying between points $\frac{(k-1)L}{N}$ and $\frac{kL}{N}$ can be denoted as $R_A'$, $R_B'$ and $R_M'$, and expressed as:

$$R_A' = p_A'{}^*\left(x_A' - \frac{(k-1)L}{N}\right) \tag{28}$$

$$R_B' = p_B'{}^*\left(\frac{k\quad L}{N} - x_B'\right) \tag{29}$$

$$R_M' = p_M'{}^*(x_D' - x_C') \tag{30}$$

The corresponding profits of sellers A, B and the platform $\pi_A'$, $\pi_B'$ and $\pi_P'$ remain consistent with the baseline model.

$$\pi_A' = (1-\rho)(R_A' + a^*R_M') \tag{31}$$

$$\pi_B' = (1-\rho)[R_B' + (1-a)^*R_M'] \tag{32}$$

$$\pi_P' = \rho(R_A' + R_B' + R_M') \tag{33}$$

We derive Proposition 6 by using the same methods as in Sections 3 and 4 and solving the following four scenarios, (1) full coverage market without probabilistic selling; (2) full coverage market with probabilistic selling; (3) partial coverage market without probabilistic selling; (2) partial coverage market with probabilistic selling, and their boundary conditions.

**Proposition 6.** *With N sellers evenly distributed on the Salop circle,*

(1) *When $t \le \frac{4N^2U}{3L^2}$, the market is fully covered. The platform does not offer probabilistic selling products.*
(2) *When $t > \frac{4N^2U}{3L^2}$, the market is partially covered. If condition (34) is satisfied, the platform opts to offer probabilistic selling products and its profits is increasing in the number of sellers in the market ($N$). Otherwise, the platform does not offer probabilistic selling products.*

$$\begin{cases} \frac{1}{4} < a \le \frac{1}{2} \ \& \text{ and } \& \ \frac{12N^2U}{L^2+4aL^2+4a^2L^2} < t < -\frac{N^2U}{-aL^2+a^2L^2} \\ \frac{1}{2} < a < \frac{3}{4} \ \& \text{ and } \& \ \frac{12N^2U}{9L^2-12aL^2+4a^2L^2} < t < -\frac{N^2U}{-aL^2+a^2L^2} \end{cases} \tag{34}$$

See the appendix for detailed proof.

From Proposition 6, we draw two key conclusions. First, the full coverage market parameter condition is satisfied as long as the number of firms in the market is large enough. Second, when the market is partially covered, as $N$ increases, the platform benefits more from the probabilistic selling products. If we further assume a fixed cost, denoted as $F$, to introduce the probabilistic selling products, the platform will only offer the probabilistic selling products when the number of sellers in the market $N$ is neither too small nor too large.

Due to $\frac{\partial N\rho R_M'^*}{\partial N} > 0$, the equation $N\rho R_M'^* = F$ has a unique solution which is the lower bound of $N$ to introduce probabilistic selling products. We denote it as $N_1$. From the partial market coverage condition (34), we derive the upper bound of $N$ to introduce probabilistic selling products. We denote it as $N_2$.

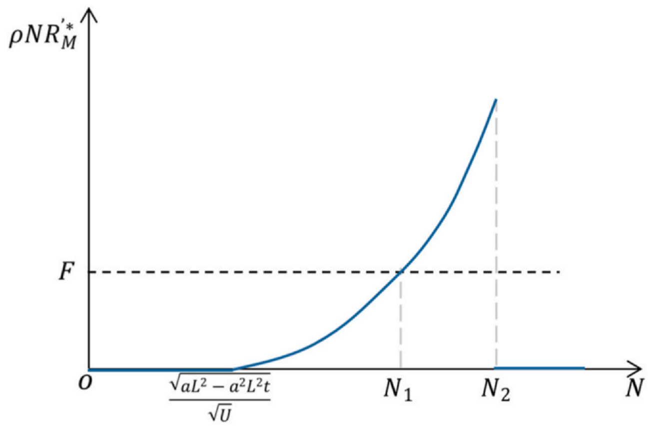



**Fig. 3.** The platform's profit in a market with N sellers.

$$N_2 = \begin{cases} \frac{\sqrt{L^2t + 4aL^2t + 4a^2L^2t}}{2\sqrt{3U}} & if \quad \frac{1}{4} < a \le \frac{1}{2} \\ \frac{\sqrt{9L^2t - 12aL^2t + 4a^2L^2t}}{2\sqrt{3U}} & if \quad \frac{1}{2} < a < \frac{3}{4} \end{cases} \tag{35}$$

Corollary 4 and Graph 3 show the results.

**Corollary 4.** *Suppose there is a fixed cost $F$ to introduce the probabilistic selling of products. When $N_1 < N < N_2$, the platform ops to offer the probabilistic selling products.*

The intuition behind the upper bound $N_2$ is that, as $N$ grows, product differentiation shrinks, since the distance a consumer has to travel to the nearest two firms shrinks. As we proved in Proposition 4, a decrease in product differentiation in the presence of probabilistic selling products increases market coverage of deterministic selling products and decreases coverage of probabilistic selling products. Applying that result, it is easy to see that the lower bound on $t$ that ensures a positive measure of market coverage for probabilistic products cannot be attained for $N$ large enough.

## 6. Conclusion

The introduction of probabilistic selling mechanisms represents a significant innovation within the evolving digital economy. It is widely used for hotel reservations and car rental businesses. This paper applies the Salop circular city model to investigate the impact of probabilistic selling on the digital platform and the sellers' profits. We find that, under specific conditions where the number of sellers is limited, baseline consumption utility is constrained, and product differentiation is moderate, the market is in partial coverage and probabilistic selling products with moderate mixing probabilities benefit the platform and the sellers simultaneously. This stems from the platform's ability to introduce probabilistic selling products to tap into new market segments without reducing prices or market coverage in the existing deterministic selling product market. Additionally, when probabilistic selling products increase firm and platform profits, they also increase consumer welfare, and so represent a Pareto improvement in the market outcome.

The aforementioned conditions align with key characteristics of the travel industry's current landscape and the preferences of certain consumer groups. First, the travel product market's competitive intensity is naturally limited by factors such as airport capacity and traffic control. Second, the ongoing impact of the COVID-19 pandemic has translated into reduced baseline consumption utility in the travel industry. Third, probabilistic selling is particularly suited for consumers with moderate product differentiation needs, encompassing individuals with budget constraints and flexible travel plans. Consequently, our findings advocate for probabilistic selling products as a crucial avenue to engage specific client groups and foster the rejuvenation of the travel sector in the post-COVID-19 pandemic era.

This study also opens up several avenues for future research. First, we do not consider the quality differences among sellers. The model does not address whether the platform would opt for probabilistic selling when competing low-cost alternatives are available. Second, we focus on the probabilistic selling behavior of a monopolistic platform and do not address competition between digital platforms. Future research could investigate how the platforms' decisions on probabilistic selling may change when sellers choose to sell tickets on different platforms.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Appendix. : Proof of Propositions

Proof of proposition 1:

Plugging Eq. (11) into Eqs. (5) to (6), we get:

$$R_A = \frac{p_A(-4p_A + 4p_B + L^2t)}{2Lt} \tag{A1}$$

$$R_B = \frac{p_B(4p_A - 4p_B + L^2t)}{2Lt} \tag{A2}$$

Solving the first-order conditions, $\frac{\partial \pi_A}{\partial p_A} = 0$ and $\frac{\partial \pi_B}{\partial p_B} = 0$, simultaneously, we obtain the equilibrium prices for both sellers as follows:

$$p_A^* = p_B^* = \frac{L^2t}{4} \tag{A3}$$

$p_A^*$ and $p_B^*$ are the unique Nash equilibrium solutions (satisfying second-order conditions $\frac{\partial^2 \pi_A}{\partial p_A^2} < 0$, $\frac{\partial^2 \pi_B}{\partial p_B^2} < 0$). Substituting $p_A^*$ and $p_B^*$ into Eq. (11) (A1) and (A3), we get the equilibrium consumers' strategies and revenue:

$$\bar{x}^* = \frac{L}{4} \tag{A4}$$

$$R_A^* = R_B^* = \frac{L^3 t}{8} \tag{A5}$$

From Eq. (10), we find the platform's expected profit is:

$$\pi_P^* = \rho(R_A^* + R_B^*) = \frac{1}{4}L^3 t\rho \tag{A6}$$

It is evident from Eq. (A6) that $\frac{\partial \pi_P^*}{\partial t} > 0$. Therefore, $\pi_P^*$ increases with the increase of $t$.

Full market coverage requires that the consumer at $\bar{x}^* = \frac{L}{4}$ has non-negative utility, satisfying the condition:

$$0 < t \leq \frac{16U}{5L^2} \tag{A7}$$

Proof of proposition 2:

Plugging Eqs. (12) and (13) into Eqs. (5) to (7), we get:

$$R_A = \frac{p_A[4p_A - 4p_M + (-1+a)L^2 t]}{2(-1+a)Lt} \tag{A8}$$

$$R_B = \frac{p_B(-4p_B + 4p_M + aL^2 t)}{2aLt} \tag{A9}$$

$$R_M = \frac{2p_M[-p_B + a(-p_A + p_B) + p_M]}{(-1+a)aLt} \tag{A10}$$

Case 1:

Solving the first-order conditions, $\frac{\partial \pi_A}{\partial p_A} = 0$ and $\frac{\partial \pi_B}{\partial p_B} = 0$, simultaneously, we get:

$$p_A = \frac{1}{8}[4(1+a)p_M - (-1+a)L^2 t] \tag{A11}$$

$$p_B = p_M - \frac{ap_M}{2} + \frac{1}{8}aL^2 t \tag{A12}$$

Plugging in Eqs. (A11) and (A12) and solving the first-order condition, $\frac{\partial \pi_P}{\partial p_M} = 0$, we obtain the equilibrium prices as follows:

$$p_A^* = p_B^* = p_M^* = \frac{L^2 t}{4} \tag{A13}$$

Case 2:

Solving the first-order condition, $\frac{\partial \pi_P}{\partial p_M} = 0$, we get:

$$p_M = a(p_A - p_B) + p_B \tag{A14}$$

Plugging in Eq. (A14), and solving the first-order conditions $\frac{\partial \pi_A}{\partial p_A} = 0$, $\frac{\partial \pi_B}{\partial p_B} = 0$, simultaneously, we obtain the equilibrium prices as follows:

$$p_A^* = p_B^* = p_M^* = \frac{L^2 t}{4} \tag{A15}$$

Case 3:

Solving the first-order conditions, $\frac{\partial \pi_A}{\partial p_A} = 0$, $\frac{\partial \pi_B}{\partial p_B} = 0$ and $\frac{\partial \pi_P}{\partial p_M} = 0$, simultaneously, we obtain the equilibrium prices as follows:

$$p_A^* = p_B^* = p_M^* = \frac{L^2 t}{4} \tag{A16}$$

Furthermore, these prices are unique Nash equilibrium solutions and satisfy the second-order conditions $\frac{\partial^2 \pi_A}{\partial p_A^2} < 0$, $\frac{\partial^2 \pi_B}{\partial p_B^2} < 0$ and $\frac{\partial^2 \pi_P}{\partial p_M^2} < 0$.

Substituting the equilibrium prices into Eqs. (12) and (13), we get:

$$x_A^* = x_B^* = x_C^* = x_D^* = \frac{L}{4} \tag{A17}$$

Proof of proposition 3:

Plugging Eqs. (14) and (15) into Eqs. (5) and (6), we get:

$$R_A = \frac{2p_A\sqrt{-p_A + U}}{\sqrt{t}} \tag{A18}$$

$$R_B = \frac{2p_B\sqrt{t(-p_B + U)}}{\sqrt{t}} \tag{A19}$$

Solving the first-order conditions, $\frac{\partial \pi_A}{\partial p_A} = 0$ and $\frac{\partial \pi_B}{\partial p_B} = 0$, simultaneously, we obtain the equilibrium prices for both sellers as follows:

$$p_A^* = p_B^* = \frac{2U}{3} \tag{A20}$$

$p_A^*$ and $p_B^*$ are the unique Nash equilibrium solutions (satisfying second-order conditions $\frac{\partial^2 \pi_A}{\partial p_A^2} < 0, \frac{\partial^2 \pi_B}{\partial p_B^2} < 0$). Substituting $p_A^*$ and $p_B^*$ into Eq. (14) (15) (A18) and (A19), we get the equilibrium consumers' strategies and revenue:

$$x_A^* = \frac{\sqrt{U}}{\sqrt{3}\sqrt{t}} \tag{A21}$$

$$x_B^* = \frac{L}{2} - \frac{\sqrt{U}}{\sqrt{3}\sqrt{t}} \tag{A22}$$

$$R_A^* = R_B^* = \frac{4U^{3/2}}{3\sqrt{3}\sqrt{t}} \tag{A23}$$

From Eq. (10), we find the platform's expected profit is:

$$\pi_P^* = \rho(R_A^* + R_B^*) = \frac{8U\sqrt{U}\rho}{3\sqrt{3}\sqrt{t}} \tag{A24}$$

It is evident from Eq. (A24) that $\frac{\partial \pi_P^*}{\partial t} < 0$ and $\frac{\partial \pi_P^*}{\partial U} > 0$. Therefore, $\pi_P^*$ increases with the decrease of $t$ or the increase of $U$.

For partial market coverage, it requires that $x_A^* < x_B^*$. We get the parameter condition:

$$t > \frac{16U}{3L^2} \tag{A25}$$

Proof of proposition 4:

Plugging Eqs. (14)-(17) into Eqs. (5) to (7), we get:

$$R_A = \frac{2p_A\sqrt{-p_A + U}}{\sqrt{t}} \tag{A26}$$

$$R_B = \frac{2p_B\sqrt{-p_B + U}}{\sqrt{t}} \tag{A27}$$

$$R_M = \frac{2p_M\sqrt{-4p_M - aL^2t + a^2L^2t + 4U}}{\sqrt{t}} \tag{A28}$$

From the first-order conditions below, we notice the selling prices $p_A$, $p_B$ and $p_M$ can be solved independently. Therefore, different timing does not influence the equilibrium results.

$$\frac{\partial \pi_A}{\partial p_A} = \frac{(3p_A - 2U)(-1 + \rho)}{\sqrt{t}\sqrt{-p_A + U}} \tag{A29}$$

$$\frac{\partial \pi_B}{\partial p_B} = \frac{(3p_B - 2U)(-1 + \rho)}{\sqrt{t}\sqrt{-p_B + U}} \tag{A30}$$

$$\frac{\partial \pi_P}{\partial p_M} = \frac{2(-6p_M - aL^2t + a^2L^2t + 4U)\rho}{\sqrt{t}\sqrt{-4p_M - aL^2t + a^2L^2t + 4U}} \tag{A31}$$

We obtain the equilibrium prices as follows:

$$p_A^* = p_B^* = \frac{2U}{3} \tag{A32}$$

$$p_M^* = \frac{1}{6}(-aL^2t + a^2L^2t + 4U) \tag{A33}$$

Furthermore, these prices are unique Nash equilibrium solutions and satisfy the second-order conditions $\frac{\partial^2 \pi_A}{\partial p_A^2} < 0$, $\frac{\partial^2 \pi_B}{\partial p_B^2} < 0$ and $\frac{\partial^2 \pi_P}{\partial p_M^2} < 0$.

Substituting the results into Eqs. (14) - (17), we get:

$$x_A^* = \frac{\sqrt{U}}{\sqrt{3}\sqrt{t}},\ x_B^* = \frac{L}{2} - \frac{\sqrt{U}}{\sqrt{3}\sqrt{t}} \tag{A34}$$

$$x_C^* = \frac{1}{2}(1-a)L - \frac{\sqrt{12U - 3(1-a)aL^2t}}{6\sqrt{t}},\ x_D^* = \frac{1}{2}(1-a)L + \frac{\sqrt{12U - 3(1-a)aL^2t}}{6\sqrt{t}} \tag{A35}$$

Proof of Corollary 2:
Without probabilistic selling, plugging in (A20) - (A22), we obtain the consumer surplus as follows:

$$CS_1 = 2\int_0^{x_A^*}(U - tx^2 - p_A^*)dx + 2\int_{x_B^*}^{\frac{L}{2}}[U - t\left(\frac{L}{2}-x\right)^2 - p_B^*]dx = \frac{8U\sqrt{tU}}{9\sqrt{3}\,t} \tag{A36}$$

Plugging (A23) into Eqs. (8) - (10), we get the profits of both sellers and the platform:

$$\pi_{A1} = \pi_{B1} = \frac{4U^{\frac{3}{2}}(1-\rho)}{3\sqrt{3t}} \tag{A37}$$

$$\pi_{P1} = \frac{8U\sqrt{U}\rho}{3\sqrt{3}\sqrt{t}} \tag{A38}$$

With probabilistic selling, plugging in (A32) - (A35), we obtain the consumer surplus as follows:

$$\begin{aligned} CS_2 &= 2\int_0^{x_A^*}(U - tx^2 - p_A^*)dx + 2\int_{x_B^*}^{\frac{L}{2}}\left[U - t\left(\frac{L}{2}-x\right)^2 - p_B^*\right]dx + 2\int_{x_C^*}^{x_D^*}\{a(U - tx^2) + (1-a)\left[U - t\left(\frac{L}{2}-x\right)^2\right] - p_M^*\}dx \\ &= \frac{8U^{\frac{3}{2}} + ((-1+a)aL^2t + 4U)^{\frac{3}{2}}}{9\sqrt{3t}} \end{aligned} \tag{A39}$$

The profits of both buyers and the platform are:

$$\pi_{A2} = -\frac{1}{3\sqrt{3t}}\left[4U^{\frac{3}{2}} + (-1+a)a^2L^2t\sqrt{(-1+a)aL^2t + 4U} + 4aU\sqrt{(-1+a)aL^2t + 4U}\right](\rho - 1) \tag{A40}$$

$$\pi_{B2} = \frac{1}{3\sqrt{3t}}\{-4U^{\frac{3}{2}} + [(-1+a)^2aL^2t + 4(-1+a)U]\sqrt{(-1+a)aL^2t + 4U}\}(\rho - 1) \tag{A41}$$

$$\pi_{P2} = \frac{\left[8U^{\frac{3}{2}} + (-1+a)aL^2t\sqrt{(-1+a)aL^2t + 4U} + 4U\sqrt{(-1+a)aL^2t + 4U}\right]\rho}{3\sqrt{3t}} \tag{A42}$$

From Eqs. (A36) to (A42), we derive the changes in different participants' welfare:

$$\Delta CS = CS_2 - CS_1 = \frac{[(-1+a)aL^2t + 4U]^{\frac{3}{2}}}{9\sqrt{3t}} \tag{A43}$$

$$\Delta\pi_A = \pi_{A2} - \pi_{A1} = -\frac{a[(-1+a)aL^2t + 4U]^{\frac{3}{2}}(\rho - 1)}{3\sqrt{3t}} \tag{A44}$$

$$\Delta\pi_B = \pi_{B2} - \pi_{B1} = \frac{(a-1)[(-1+a)aL^2t + 4U]^{\frac{3}{2}}(\rho - 1)}{3\sqrt{3t}} \tag{A45}$$

$$\Delta\pi_P = \pi_{P2} - \pi_{P1} = \frac{[(-1+a)aL^2t + 4U]^{\frac{3}{2}}\rho}{3\sqrt{3t}} \tag{A46}$$

$$\Delta SW = \Delta CS + \Delta\pi_A + \Delta\pi_B + \Delta\pi_P = \frac{4[(-1+a)aL^2 + 4U]^{\frac{3}{2}}}{9\sqrt{3t}} \tag{A47}$$

When condition (18) is satisfied, it is evident that $\Delta CS > 0$, $\Delta\pi_A > 0$, $\Delta\pi_B > 0$, $\Delta\pi_P > 0$ and $\Delta$SW $> 0$.
Proof of proposition 5:
Plugging $x_C^*$, $x_D^*$ and $p_M^*$ in Proposition 4 into Eq. (7), we get:

$$R_M^* = \frac{(-aL^2t + a^2L^2t + 4U)^{3/2}}{3\sqrt{3}\sqrt{t}} \tag{A48}$$

Suppose $0 < a \le \frac{1}{2}$. To guarantee the partial coverage market, $0 < x_A < x_C < x_D < x_B$ must be satisfied:

$$\begin{cases} \frac{12U}{L^2} < t \le \frac{16U}{L^2} \quad and -\frac{1}{2} + \frac{2\sqrt{3U}}{\sqrt{L^2 t}} < a < \frac{1}{2} \\ \frac{16U}{L^2} < t < \frac{64U}{3L^2} \quad and -\frac{1}{2} + \frac{2\sqrt{3U}}{\sqrt{L^2 t}} < a < \frac{1}{2} - \frac{1}{2}\frac{\sqrt{L^2 t - 16U}}{\sqrt{L^2 t}} \end{cases} \tag{A49}$$

Given any $t \in (\frac{12U}{L^2}, \frac{64U}{3L^2})$, it is easy to prove that:

$$\frac{\partial R_M^*}{\partial a} < 0 \tag{A50}$$

Therefore, the optimal mixing probability:

$$a^*(t) = -\frac{1}{2} + \frac{2\sqrt{3U}}{\sqrt{L^2 t}} \tag{A51}$$

$$R_M^*(a^*(t)) = \frac{[64U + L^2 t(3 - \frac{16\sqrt{3U}}{\sqrt{L^2 t}})]^{3/2}}{24\sqrt{3t}} \tag{A52}$$

Suppose $\frac{1}{2} < a < 1$. To guarantee the partial coverage market, $0 < x_A < x_C < x_D < x_B$ must be satisfied:

$$\begin{cases} \frac{12U}{L^2} < t \le \frac{16U}{L^2} \quad and \quad \frac{1}{2} < a < \frac{3}{2} - \frac{2\sqrt{3U}}{\sqrt{L^2 t}} \\ \frac{16U}{L^2} < t < \frac{64U}{3L^2} \quad and \quad \frac{1}{2} + \frac{1}{2}\frac{\sqrt{L^2 t - 16U}}{\sqrt{L^2 t}} < a < \frac{3}{2} - \frac{2\sqrt{3U}}{\sqrt{L^2 t}} \end{cases} \tag{A53}$$

Given any $t \in (\frac{12U}{L^2}, \frac{64U}{3L^2})$, it is easy to prove that:

$$\frac{\partial \quad R_M^*}{\partial a} > 0$$

Therefore, the optimal mixing probability:

$$a^*(t) = \frac{3}{2} - \frac{2\sqrt{3U}}{\sqrt{L^2 t}} \tag{A54}$$

$$R_M^*(a^*(t)) = \frac{[64U + L^2 t(3 - \frac{16\sqrt{3U}}{\sqrt{L^2 t}})]^{3/2}}{24\sqrt{3t}} \tag{A55}$$

Proof of Corollary 3:

From Eq. (A39), we know the consumer surplus as follows:

$$CS = \frac{8U^{\frac{3}{2}} + ((-1 + a)aL^2 t + 4U)^{\frac{3}{2}}}{9\sqrt{3t}} \tag{A56}$$

Suppose $0 < a \le \frac{1}{2}$. To guarantee the partial coverage market, condition (A49) must be satisfied. When $\frac{12U}{L^2} < t \le \frac{16U}{L^2} and -\frac{1}{2} + \frac{2\sqrt{3U}}{\sqrt{L^2 t}} < a < \frac{1}{2}$ or $\frac{16U}{L^2} < t < \frac{64U}{3L^2} and -\frac{1}{2} + \frac{2\sqrt{3U}}{\sqrt{L^2 t}} < a < \frac{1}{2} - \frac{1}{2}\frac{\sqrt{L^2 t - 16U}}{\sqrt{L^2 t}}$, it can be readily demonstrated that

$$\frac{\partial CS}{\partial a} < 0$$

Therefore, the optimal mixing probability $a^{`}(t) = -\frac{1}{2} + \frac{2\sqrt{3U}}{\sqrt{L^2 t}}$ is as same as $a^*(t)$.

Suppose $\frac{1}{2} < a < 1$. To guarantee the partial coverage market, condition (A53) must be satisfied. When $\frac{12U}{L^2} < t \le \frac{16U}{L^2} and \frac{1}{2} < a < \frac{3}{2} - \frac{2\sqrt{3U}}{\sqrt{L^2 t}}$ or $\frac{16U}{L^2} < t < \frac{64U}{3L^2} and \frac{1}{2} + \frac{1}{2}\frac{\sqrt{L^2 t - 16U}}{\sqrt{L^2 t}} < a < \frac{3}{2} - \frac{2\sqrt{3U}}{\sqrt{L^2 t}}$, it can be readily demonstrated that

$$\frac{\partial CS}{\partial a} > 0$$

Therefore, the optimal mixing probability $a^{`}(t) = \frac{3}{2} - \frac{2\sqrt{3U}}{\sqrt{L^2 t}}$ is as same as $a^*(t)$.

In summary, $a^*(t)$ maximizes the consumer surplus and the profits of the platform and sellers simultaneously. Obviously, it also maximizes the total social welfare.

Proof of proposition 6:

Scenario 1: the full coverage market without probabilistic selling.

Let $u'_A = u'_B$, we get the position of the indifferent consumer as

$$\bar{x}' = \frac{(-1 + 2k)L}{2N} + \frac{N(-p'_A + p'_B)}{2Lt} \tag{A57}$$

Plugging Eq. (A57) into Eqs. (28) to (29), we get

$$R'_A = \frac{Lp'_A}{2N} + \frac{Np'_A(-p'_A + p'_B)}{2Lt} \tag{A58}$$

$$R'_B = \frac{Lp'_B}{2N} + \frac{Np'_B(p'_A - p'_B)}{2Lt} \tag{A59}$$

Solving the first-order conditions, $\frac{\partial \pi'_A}{\partial p'_A} = 0$ and $\frac{\partial \pi'_B}{\partial p'_B} = 0$, simultaneously, we obtain the equilibrium prices for both sellers as follows (the unique Nash equilibrium solutions satisfying second-order conditions $\frac{\partial^2 \pi'_A}{\partial p'^2_A} < 0$, $\frac{\partial^2 \pi'_B}{\partial p'^2_B} < 0$)

$$p'^*_A = p'^*_B = \frac{L^2 t}{N^2} \tag{A60}$$

Then the revenue and platform's expected profits are

$$R'^*_A = R'^*_B = \frac{L^3 t}{2N^3} \tag{A61}$$

$$\pi'^*_A = \pi'^*_B = -\frac{L^3 t(-1 + \rho)}{2N^3} \tag{A62}$$

$$\pi'^*_P = \frac{\rho L^3 t}{N^3} \tag{A63}$$

For full market coverage, it requires that the consumer at $\bar{x}'$has non-negative utility, satisfying the condition

$$0 < t \le \frac{4N^2 U}{5L^2} \tag{A64}$$

Scenario 2: the full coverage market with probabilistic selling.

Let $u'_A = u'_M$ and $u'_B = u'_M$, we can represent different consumers' positions $x'_A$ and $x'_A$ after substituting Eqs. (24)–(26) as follows:

$$x'_C = x'_A = \frac{(-1 + 2k)L}{2N} - \frac{Np'_A - Np'_M}{2Lt(1 - a)} \tag{A65}$$

$$x'_D = x'_B = \frac{(-1 + 2k)L}{2N} + \frac{Np'_B - Np'_M}{2Lta} \tag{A66}$$

Plugging Eqs. (A65) and (A66) into Eqs. (28) to (30), we get

$$R'_A = \frac{Lp'_A}{2N} + \frac{Np'_A(p'_A - p'_M)}{2(a - 1)Lt} \tag{A67}$$

$$R'_B = \frac{Lp'_B}{2N} + \frac{Np'_B(p'_M - p'_B)}{2aLt} \tag{A68}$$

$$R'_M = \frac{Np'_M[-p'_B + a(p'_B - p'_A) + p'_M]}{2(-1 + a)aLt} \tag{A69}$$

Case 1:

Solving the first-order conditions, $\frac{\partial \pi'_A}{\partial p'_A} = 0$ and $\frac{\partial \pi'_B}{\partial p'_B} = 0$, simultaneously, we get

$$p'_A = \frac{1}{2}\left[(1 + a)p'_M - \frac{(-1 + a)L^2 t}{N^2}\right] \tag{A70}$$

$$p'_B = p'_M - \frac{ap'_M}{2} + \frac{aL^2 t}{2N^2} \tag{A71}$$

Solve the first-order condition, $\frac{\partial \pi'_p}{\partial p'_M} = 0$, we obtain the equilibrium prices as follows:

$$p'^*_A = p'^*_B = p'^*_M = \frac{L^2 t}{N^2} \tag{A72}$$

Case 2:

Solving the first-order condition, $\frac{\partial \pi'_p}{\partial p'_M} = 0$, we get

$$p'_M = a(p'_A - p'_B) + p'_B \tag{A73}$$

Plugging in Eq. (A73), and solving the first-order conditions, $\frac{\partial \pi'_A}{\partial p'_A} = 0$ and $\frac{\partial \pi'_B}{\partial p'_B} = 0$, simultaneously, we obtain the equilibrium prices as follows

$$p'^*_A = p'^*_B = p'^*_M = \frac{L^2 t}{N^2} \tag{A74}$$

Case 3:

Solving the first-order conditions, $\frac{\partial \pi'_A}{\partial p'_A} = 0$, $\frac{\partial \pi'_B}{\partial p'_B} = 0$ and $\frac{\partial \pi'_P}{\partial p'_M} = 0$, simultaneously, we obtain the equilibrium prices as follows:

$$p'^*_A = p'^*_B = p'^*_M = \frac{L^2 t}{N^2} \tag{A75}$$

Furthermore, these prices are unique Nash equilibrium solutions and satisfy the second-order conditions $\frac{\partial^2 \pi'_A}{\partial p'^2_A} < 0$, $\frac{\partial^2 \pi'_B}{\partial p'^2_B} < 0$ and $\frac{\partial^2 \pi'_P}{\partial p'^2_M} < 0$.

Substituting the equilibrium prices into Eqs. (A65) and (A66), we get

$$x'^*_A = x'^*_B = x'^*_C = x'^*_D = \frac{(2k-1)L}{2N} \tag{A76}$$

Therefore, consumers lying on the arc between points $\frac{(k-1)L}{N}$ and $x'^*_A = \frac{(-1+2k)L}{2N}$ purchase tickets A. Consumers lying on the arc between points $x'^*_B = \frac{(-1+2k)L}{2N}$ and $\frac{kL}{N}$ purchase tickets B. The market demand for probabilistic tickets is 0 and the platform will not offer probabilistic selling tickets.

It is worth noting that when $\frac{4N^2 U}{5L^2} < t \le \frac{4N^2 U}{3L^2}$, the market is still fully covered, but there is no market competition between sellers A and B. In this scenario, the equilibrium prices for tickets are $p'^*_A = p'^*_B = p'^*_M = U - \frac{L^2 t}{4N^2}$ and $x'^*_A = x'^*_B = x'^*_C = x'^*_D = \frac{(-1+2k)L}{2N}$.

Scenario 3: the partial coverage market without probabilistic selling.

Let $u'_A = 0$ and $u'_B = 0$, we derive the positions of the utilities of the marginal consumers who choose to buy tickets A and B.

$$x'_A = \frac{(-1+k)L}{N} + \frac{\sqrt{t(-p'_A + U)}}{t} \tag{A77}$$

$$x'_B = \frac{kL}{N} - \frac{\sqrt{t(-p'_B + U)}}{t} \tag{A78}$$

Plugging Eqs. (A77) and (A78) into Eqs. (28) and (29), we get

$$R'_A = \frac{p'_A\sqrt{-p'_A + U}}{\sqrt{t}} \tag{A79}$$

$$R'_B = \frac{p'_B\sqrt{-p'_B + U)}}{\sqrt{t}} \tag{A80}$$

Solving the first-order conditions, $\frac{\partial \pi'_A}{\partial p'_A} = 0$ and $\frac{\partial \pi'_B}{\partial p'_B} = 0$, simultaneously, we obtain the equilibrium prices for both sellers as follows

$$p'^*_A = p'^*_B = \frac{2U}{3} \tag{A81}$$

$p'^*_A$ and $p'^*_B$ are the unique Nash equilibrium solutions (satisfying second-order conditions $\frac{\partial^2 \pi'_A}{\partial p'^2_A} < 0$, $\frac{\partial^2 \pi'_B}{\partial p'^2_B} < 0$). Substituting $p'^*_A$ and $p'^*_B$ into Eqs. (A77) - (A80), we get the equilibrium consumers' strategies and revenue

$$x'^*_A = \frac{(-1+k)L}{N} + \frac{\sqrt{tU}}{\sqrt{3}t} \tag{A82}$$

$$x'^*_B = \frac{kL}{N} - \frac{\sqrt{tU}}{\sqrt{3}t} \tag{A83}$$

$$R'^*_A = R'^*_B = \frac{2U^{3/2}}{3\sqrt{3}\sqrt{t}} \tag{A84}$$

From Eq. (A84), we get

$$\pi'^*_P = \rho(R'^*_A + R'^*_B) = \frac{4U\sqrt{U}\rho}{3\sqrt{3}\sqrt{t}} \tag{A85}$$

For partial market coverage, it requires that consumers at $x'^*_A < x'^*_B$. We derive the parameter condition

$$t > \frac{4N^2 U}{3L^2} \tag{A86}$$

Scenario 4: the partial coverage market with probabilistic selling.

Setting $u'_A = 0$, $u'_B = 0$ and $u'_M = 0$, we can solve the positions of marginal consumers who purchase tickets A, B and probabilistic selling tickets

$$x'_A = \frac{(-1+k)L}{N} + \frac{\sqrt{t(-p'_A + U)}}{t},\ x'_B = \frac{kL}{N} - \frac{\sqrt{t(-p'_B + U)}}{t} \tag{A87}$$

$$x'_C = -\frac{aLt - kLt + \sqrt{t[(-1+a)aL^2t + N^2(-p'_M + U)]}}{Nt} \tag{A88}$$

$$x'_D = \frac{-aLt + kLt + \sqrt{t[(-1+a)aL^2t + N^2(-p'_M + U)]}}{Nt} \tag{A89}$$

Plugging Eqs. (A87) – (A89) into Eqs. (28) to (30), we get

$$R'_A = \frac{p'_A\sqrt{-p'_A + U}}{\sqrt{t}} \tag{A90}$$

$$R'_B = \frac{p'_B\sqrt{(-p'_B + U)}}{\sqrt{t}} \tag{A91}$$

$$R'_M = \frac{2\quad p'_M\sqrt{t[(-1+a)aL^2t + N^2(-p'_M + U)]}}{Nt} \tag{A92}$$

From the first-order conditions, $\frac{\partial \pi'_A}{\partial p'_A} = 0$, $\frac{\partial \pi'_B}{\partial p'_B} = 0$ and $\frac{\partial \pi'_P}{\partial p'_M} = 0$, we obtain the equilibrium prices as follows:

$$p'^*_A = p'^*_B = \frac{2U}{3} \tag{A93}$$

$$p'^*_M = \frac{2(-aL^2t + a^2L^2t + N^2U)}{3N^2} \tag{A94}$$

Furthermore, these prices are unique Nash equilibrium solutions and satisfy the second-order conditions $\frac{\partial^2 \pi'_A}{\partial p'^2_A} < 0$, $\frac{\partial^2 \pi'_B}{\partial p'^2_B} < 0$ and $\frac{\partial^2 \pi'_P}{\partial p'^2_M} < 0$.

Substituting the results into Eqs. (28) to (30), we get

$$R'^*_A = R'^*_B = \frac{2U^2}{3\sqrt{3}\sqrt{tU}} \tag{A95}$$

$$R'^*_M = \frac{4[t(-aL^2t + a^2L^2t + N^2U)]^{3/2}}{3\sqrt{3}N^3t^2} \tag{A96}$$

By verifying the condition of market incomplete coverage, i.e., $\frac{(-1+k)L}{N} < x'_A < x'_C < x'_D < x'_B < \frac{kL}{N}$, it can be deduced that

$$\begin{cases} \frac{1}{4} < a \le \frac{1}{2} \text{ and } \frac{12N^2U}{L^2 + 4aL^2 + 4a^2L^2} < t < -\frac{N^2U}{-aL^2 + a^2L^2} \\ \frac{1}{2} < a < \frac{3}{4} \text{ and } \frac{12N^2U}{9L^2 - 12aL^2 + 4a^2L^2} < t < -\frac{N^2U}{-aL^2 + a^2L^2} \end{cases} \tag{A97}$$

It is easy to be observed that offering the probabilistic selling tickets does not affect the expected revenue of tickets A and B. Therefore, offering the probabilistic selling tickets enhance the digital platform's profit by $N\rho R'^*_M$. We can verify that $N\rho R'^*_M$ is positive and increasing in $N$ when the condition (A97) is satisfied.

$$\frac{\partial N\rho R'^*_M}{\partial N} \ge 0 \tag{A98}$$

In summary of the aforementioned four scenarios, we can deduce the following:

When $t \le \frac{4N^2U}{3L^2}$, that is $N \ge \frac{\sqrt{3L^2t}}{2\sqrt{U}}$, the market is fully covered and $R'^*_M = 0$. Thus, the platform chooses not to offer probabilistic selling tickets.

When $t > \frac{4N^2U}{3L^2}$, that is $N < \frac{\sqrt{3L^2t}}{2\sqrt{U}}$, the market is partially covered. The platform chooses to offer probabilistic selling and its profit is increasing in the number of sellers in the market ($N$) if conditions (A97) are met.

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